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Estimation of heuristic switching in behavioral macroeconomic models

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ABSTRACT

This paper addresses the issue of empirical validation of macroeconomic models with behavioral heuristics and a nonlinear switching mechanism. Heuristic switching is an important feature of modeling strategy since it uses simple decision rules of boundedly rational heterogeneous agents. The simulation study shows that the proposed simulated maximum likelihood method well identifies behavioral effects that remain hidden under standard econometric approaches. In the empirical application, we estimate the structural and behavioral parameters of the US economy. We are specifically able to reliably identify the intensity of choice that governs the models' nonlinear dynamics. Our empirical results thus lay the foundation for studying monetary and fiscal policy in a behavioral macroeconomic framework.

1. Introduction

The implications of the departures from fully rational agents have become one of the central issues in macroeconomic modeling. As suggested by Simon (1972), economic agents act under bounded rationality (BR) instead of perfectly processing all information using unlimited cognitive capacities. In most situations, human behavior follows simple decision rules known as behavioral heuristics that have proven to be suitable in the past.

The theoretical literature has long attempted to formalize deviations from rational expectations (RE) and to describe the decision making of boundedly rational heterogeneous agents. One such possible macroeconomic framework is a business cycle model with heuristics and a nonlinear switching mechanism (De Grauwe, 2010). However, this approach has not yet been satisfactorily reflected in the empirical literature, and it is often difficult to identify certain behavioral effects in the current nonlinear macroeconomic models. This has important implications regarding monetary and fiscal policy interventions because business cycle dynamics and the volatility in key economic variables like inflation might heavily depend on turnovers in the agents' decision process under BR. For example, Cornea-Madeira et al. (2019) state that the degree of be-

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havioral heterogeneity varies considerably over time and that monetary policy should be aware of potentially destabilizing heterogeneous expectations. Hommes et al. (2019) argue that in a behavioral environment under a flexible central bank inflation targeting regime, i.e., given a simultaneous attempt to stabilize both the output gap and the inflation rate, the volatility of the latter decreases only up to a certain point but increases after this threshold is passed. We thus propose a new econometric method that allows estimating these models, especially to identify important behavioral parameters crucial to monetary policy under BR.

As a point of departure, two very recent key contributions that we directly build on are De Grauwe and Ji (2020) and Hommes et al. (2019). De Grauwe and Ji (2020) study the effects of structural reforms regarding price flexibility and labor market rigidities in a hybrid heuristic switching New-Keynesian model (NKM), i.e., a model framework with leads and lags. The authors show that their model accounts for the empirical observation of a non-Gaussian distributed and highly persistent output gap. Hommes et al. (2019) consider a forward-looking model similar to that by De Grauwe and Ji (2020) but with a higher number of forecasting heuristics to choose from. The microfoundations of the corresponding heuristics based on experiments are presented in Anufriev and Hommes (2012). A mechanism for dynamic endogenous switching of economic agents between groups applying particular heuristics is the multinomial logistic model (Brock and Hommes, 1997). This approach has become a prominent feature of models in financial economics over the last two decades (Dieci and He, 2018; Franke and Westerhoff, 2012; Hommes, 2006, among others) and recently also in dynamic stochastic general equilibrium (DSGE) models.

Although the related econometric research has made significant progress in the last decade, using these types of models with the empirical data remains challenging. First, the choice of empirical variables is not straightforward, as standard econometric tools assume the stationarity of the input data that is often violated for macroeconomic time series. Second, because of a relatively large number of unknown parameters, some of the coefficients might not be identified. This requires new tools for parameter estimation (Del Negro et al., 2007; Kleibergen and Mavroeidis, 2014; Kulish and Pagan, 2017). Third, although nonlinear models can be represented in the state-space form, analytical solutions of the joint probability density seldom exist to allow for, e.g., maximum likelihood inference. Finally, because of the nonlinear structure, "a possibly non-monotonic likelihood surface ... tends to be rugged making it challenging to find a global optimum" (Lux and Zwinkels, 2018). The presented challenges and rapidly increasing operational capacity of computers have motivated simulation-based estimation methods that make empirical inference possible for analytically intractable models.

We propose a simulated maximum likelihood (SML) estimation method, which addresses the four abovementioned issues. Our main contribution lies in the modification of the univariate SML estimator and its transfer from financial econometrics (Altissimo and Mele, 2009; Kristensen and Shin, 2012; Kukacka and Barunik, 2017; Lee and Song, 2015) to macroeconomic optimization. We first pursue a simulation study to test its performance in a controlled environment. Compared to the standard maximum likelihood estimator (MLE), the SML technique numerically approximates each observation's conditional density via a standard kernel method. The derivation of the SML estimator is then similar to the MLE. Through a kernel approximation, the method elegantly bypasses the distributional assumptions of the MLE and the Bayesian approach, or the moment selection problem of the simulated method of moments (SMM). It also leads to a smooth surface of the simulated likelihood function, which generally supports optimization search. Finally, we illustrate the first empirical application of the multivariate SML for the estimation of a heuristic switching NKM.

As an empirical novelty, we report statistically significant estimates for the intensity of choice of macroeconomic forecasters forming expectations about the future output gap and inflation rate, which is a parameter that governs the multinomial logistic switching mechanism. The intensity of choice drives the agents' sensitivity regarding the individual heuristics, and its value is crucial for the switching process from one group to another. Note that this parameter is crucial for stability in nonlinear models with regime-switching, which gives rise to the possibility of bifurcation (Gaunersdorfer et al., 2008). This is of serious concern when these macroeconomic models are used for policy consulting. For instance, Cornea-Madeira et al. (2019) provide a summary of observations taken from the literature indicating that a unique stationary solution under RE does not necessarily prevail under BR. Instead, multiple equilibria and complex dynamics might occur when applying the multinomial logistic switching approach. Using simulations, we first show that the SML method can identify this important parameter for the intensity of choice. We then successfully identify the empirical value of the intensity of choice for the US economy, which is approximately between 1.4 and 1.5. We thus find the switching process to be less intense, i.e., with a low tendency towards coordinating all agents to adopt a dominant forecasting strategy. We also estimate all additional behavioral parameters together with structural parameters, especially the Taylor rule coefficients.

To summarize, the main contribution of this paper is the following. First and foremost, we introduce a new powerful estimation method into behavioral macroeconometrics and verify its performance for use in practice. Second, from an econometric perspective, we modify its univariate version and transfer it from financial econometrics to a multivariate macroeconomic setup. Third, methodologically, we bridge the gap between macro lab experiments and empirical estimation of behavioral macroeconomic models. Finally, from an empirical and policy-making perspective, we estimate the behavioral parameters of the US economy. In particular, we are able to reliably identify the intensity of choice for macroeconomic forecasters of output and inflation, which can be seen as crucial when conducting specific policy interventions.

This paper proceeds as follows. The next section provides an overview of the related literature. Section 3.2 describes the heuristic switching NKM. In Section 4, we introduce the SML estimation method modified for multivariate macroeconomic models. Section 5 presents a Monte Carlo simulation study of the suggested estimation framework's performance before we discuss the empirical results in Section 6. Finally, Section 7 concludes the paper. The technical details and additional results are relegated to a regular Appendix and an Online Appendix.

2. Related literature

The analysis of behavioral macroeconomic models has become prominent over the last decade; see Franke and Westerhoff (2017), Dilaver et al. (2018), Hommes (2021) for excellent overviews. A subclass of these behavioral frameworks can be identified as heuristic switching models. Although this type of model has been considered in financial economics for more than two decades (Brock and Hommes, 1997), in macroeconomics, such a modeling approach is rather new (De Grauwe, 2010; 2011; De Grauwe and Ji, 2019).

Heuristics represent simple rules of behavior that originate from the fact that the structure of the economy is observable, but the interactions between relevant variables such as output and inflation are barely comprehensible (Munier et al., 1999). Such boundedly rational behavior in response to a lack of complete information is based on habits, imitation and/or procedural optimization (Day et al., 1991). Related research questions have already drawn interest in the field of experimental economics (Anufriev et al., 2018a).

Various studies reveal that switching between forecasting heuristics based on the multinomial logistic model can indeed be observed within a laboratory environment; we refer to Assenza et al. (2014) and Hommes (2021) for literature reviews that include corresponding evidence in a DSGE context. In the same vein, Anufriev et al. (2018b) incentivize studying the empirical validity of heuristic switching models outside a financial market framework. In our paper, we follow their footsteps by adopting a similar version of the 4-type heuristic switching model approach presented in their study. This is done in line with Hommes et al. (2019) by focusing on a DSGE model.

The empirical estimation of behavioral models has rapidly gained attention in macroeconomics, which naturally includes heuristic switching models.[1] Among the first to estimate such a macroeconomic system are Liu and Minford (2014) who employ the indirect inference method (Gourieroux et al., 1993). The authors primarily test the model by De Grauwe (2010) against US data and strongly reject the behavioral model in favor of the RE model. However, a set of numerical parameter values via indirect estimation is also provided with a low estimated value (0.85) for the intensity of choice being reported. The estimates' standard errors are not available, which calls into question the statistical significance of the true parameters.

Several previous macroeconomic studies have had difficulties estimating the intensity of choice for macroeconomic forecasters of output and inflation because heuristic switching models exhibit a substantially more complex nonlinear structure than linearized DSGE models under RE (Lux and Zwinkels, 2018). Grazzini et al. (2017) estimate the model by De Grauwe (2012) via Bayesian techniques. They report a limited effect of the intensity of choice on the model behavior, as the corresponding posterior distribution does not depart significantly from the prior distribution. Other proponents of the Bayesian estimation of a heuristic switching NKM are Deák et al. (2017) who, however, fix this parameter to one. Jang and Sacht (2016, 2021) apply the SMM but also leave the intensity of choice unestimated for the De Grauwe (2011) model. Instead, they follow a grid calibration strategy that compares the results while varying the intensity of choice between 0.1 and 100.

A rare exception is given by Cornea-Madeira et al. (2019), who report a significant estimate for the intensity of choice parameter. However, in their study, the authors apply a single-equation estimation of the New Keynesian Phillips curve, which accounts for switching between heterogeneous firms. Two very recent studies are by Özden (2021) and Fischer (2022). Both authors report a significant estimate for the switching parameter when estimating a BR NKM while considering the zero lower bound for the nominal interest rate via Bayesian techniques. However, in comparison to our study, the heuristics in both studies are not supported by microevidence through laboratory experiments but rather imposed ex post in the spirit of Evans and Honkapohja (2001).

3. Expectation formation in the heuristic switching model

3.1. The core structure of the baseline NKM

The baseline NKM represents a realistic macroeconomic model with multivariate observables. It allows the study of the performance of the SML estimator in a more complex framework than the univariate applications in finance.[2] The baseline NKM reads as follows:

$$y_t = \frac{1}{1+\chi}\tilde{E}_t^j y_{t+1} + \frac{\chi}{1+\chi} y_{t-1} - \tau(r_t - \tilde{E}_t^j \pi_{t+1}) + \varepsilon_{y,t}, \tag{1}$$

[1] The growing interest in behavioral macroeconomic models has recently been accompanied by novel approaches to their empirical validation (Barde, 2017; 2020; 2022; Barde and van der Hoog, 2017; Dyer et al., 2022; Guerini and Moneta, 2017; Lamperti, 2018b; Lamperti et al., 2018; Martinoli et al., 2022; Vandin et al., 2022). Lux and Zwinkels (2018), Fagiolo et al. (2019) provide excellent surveys of these attempts.

[2] Examples are given by Kristensen and Shin (2012) and Kukacka and Barunik (2017). Kristensen and Shin (2012) estimate the following three types of models: the short-term interest rate model by Cox et al. (1985); the jump-diffusion model of daily stock returns by Andersen et al. (2002); and generic Markov decision processes. Kukacka and Barunik (2017) estimate the Brock and Hommes (1998) heterogeneous agent model. An introduction to the baseline NKM is presented in Galí (2015).

$$\pi_t = \frac{\nu}{1+\alpha\nu}\tilde{E}_t^j \pi_{t+1} + \frac{\alpha}{1+\alpha\nu}\pi_{t-1} + \kappa y_t + \varepsilon_{\pi,t}, \tag{2}$$

$$r_t = \phi_r r_{t-1} + (1-\phi_r)(\phi_\pi \pi_t + \phi_y y_t) + \varepsilon_{r,t}, \tag{3}$$

where superscripts $j = \{\text{BR, RE}\}$ refer to the BR and the RE model specification, respectively. Therefore, we distinguish between the BR NKM and the hybrid RE NKM. The corresponding expectations operator is $\tilde{E}_t^j$, which is explicitly specified below for both models.

All variables are given in quarterly magnitudes. In Eq. (1), the dynamic IS curve results from the intertemporal optimization of consumption and saving, which leads to consumption smoothing. Parameter $\tau \geq 0$ denotes the inverse intertemporal elasticity of substitution in consumption behavior. Equation (2) represents the New Keynesian Phillips curve, where the output gap (y_t) acts as the driving force for inflation dynamics that originates from monopolistic competition and Calvo-type sticky prices. The slope of the New Keynesian Phillips curve is given by parameter $\kappa \geq 0$. Parameter ν represents the discount factor ($0 < \nu < 1$). Intrinsic persistence is incorporated into the demand and supply equations using the parameters for habit formation $0 \leq \chi \leq 1$ and price indexation $0 \leq \alpha \leq 1$, respectively. According to the Taylor rule (3) with interest rate smoothing ($\phi_r \geq 0$), the monetary authority reacts directly to contemporaneous movements in output ($\phi_y \geq 0$) and inflation ($\phi_\pi \geq 0$). We assume that the exogenous driving forces in the model variables follow idiosyncratic shocks $\varepsilon_{\{y,\pi,r\},t}$, which are independent and identically distributed (i.i.d.) with mean zero and variances $\sigma^2_{\{y,\pi,r\}}$.

3.2. The model under bounded rationality

For the BR specification of the model, we directly follow the modeling approach by Hommes et al. (2019) where, at its core, the purely forward-looking version of the baseline NKM is considered, i.e., with the intrinsic persistence parameters χ, α, and ϕ_r all set to zero. Regarding the expectation formation process, specific heuristics are applied, which are discussed below. The empirical microfoundation of these heuristics is discussed in Anufriev and Hommes (2012) who reinvestigate learning-to-forecast experiments by Hommes et al. (2005, 2008) given a standard asset pricing model. The authors report that the heuristics being used account for the stylized facts of a slow (almost) monotonic convergence, persistent oscillations with almost constant amplitude, and/or large initial but then dampening oscillations observed in the dynamics of their model. Hommes et al. (2019) confirm these findings based on their own experimental macroeconomic setup where all three types of heuristics are supported by experimental evidence. The authors also report that their heuristic switching model predicts subjects' forecasts better than that under RE.

The following heuristics are considered in Hommes et al. (2019):

$$E^{ADA}x_{t+1} = \eta x_{t-1} + (1-\eta)E^{ADA}x_t, \tag{4}$$

$$E^{TR}x_{t+1} = x_{t-1} + \iota(x_{t-1} - x_{t-2}), \tag{5}$$

$$E^{LAA}x_{t+1} = \mu(x^{av}_{t-1} + x_{t-1}) + (x_{t-1} - x_{t-2}). \tag{6}$$

For an in-depth discussion of these heuristics, we refer directly to Anufriev and Hommes (2012, pp. 45–46). Similarly to Hommes et al. (2019), we apply these heuristics with respect to both the output gap and inflation rate expectations, which implies that $x = \{y, \pi\}$. Agents therefore sort themselves into three groups of output gap and inflation forecasters. Based on the *adaptive* (*ADA*) heuristic (4), future expectation results from the weighted sum of the previous realization in x and agents' known past prediction denoted by $E^{ADA}x_t$ with $0 \leq \eta \leq 1$. In the polar case $\eta = 1$, the expression stands for a static/naïve expectation formation process. In the *trend-following* (*TR*) heuristic (5), the past realization is considered while this forecasting rule follows the direction of the last change in x given by the term $(x_{t-1} - x_{t-2})$. The corresponding parameter of extrapolation denoted by $\iota \geq 0$ is crucial for observing specific patterns in the variable's dynamics. Finally, in the *learning anchoring and adjustment* (*LAA*) heuristic, the last change in x given by $(x_{t-1} - x_{t-2})$ is further extrapolated from an anchor learned through the sum of the average of all observations up to time $t-1$, denoted x^{av}_{t-1}, and the last available realization. The corresponding anchoring parameter is given by $\mu \geq 0$. Note that we consider a generalization of the *LAA* heuristic specification found in Hommes et al. (2019) with a freely estimated parameter instead of an implicit constant.[3] Therefore, we account for behavioral parameters in all heuristics under investigation, which allows for more flexibility in applying these rules-of-thumb on a macro scale.[4]

[3] In Hommes et al. (2019), the first term in (6) actually reads as $\frac{x^{av}_{t-1}+x_{t-1}}{2}$ because the authors consider the average of the sum of x^{av}_{t-1} and x_{t-1} as the anchor. In our case, the equivalent expression would be $2\mu\frac{x^{av}_{t-1}+x_{t-1}}{2}$. Since the estimation procedure is not affected by the scale parameter 2, our *LAA* heuristic (6) holds for convenience.

[4] More sophisticated expectation formation concepts than simple one-period backward-looking forecasting heuristics can potentially also be accommodated, e.g., AR or VAR forecasting models (Chung and Xiao, 2013; Milani, 2011) used by the agents, recursive least squares learning (Branch and McGough, 2018; Evans and Honkapohja, 1996), constant gain learning algorithms (Fischer, 2022), genetic algorithm learning (Hommes et al., 2017), or even more

For the BR specification, the switching from one group to the other is based on the multinomial logistic model. The expression for the market forecast regarding the output gap and the inflation rate across the three groups is given by

$$\tilde{E}_t^{BR} x_{t+1} = \sum_{i=1}^{3} (\alpha_{x,t}^{k\{i\}} \cdot E_t^{k\{i\}} x_{t+1}), \tag{7}$$

with $k = \{ADA, TR, LAA\}$. The probability $\alpha_{x,t}^k$ represents stochastic behavior by the agents who adopt a particular forecasting heuristic. Thus, $\alpha_{x,t}^k$ can be interpreted as the probability of being an adaptive forecaster, a trend-follower, or an anchored forecaster with respect to the development of the output gap and inflation rate at time t. The selection of the forecasting heuristics (4)–(6) depends on the forecast performance of each group U_t^k, which is given by the most recent squared forecasting error. The utility of the forecast performance can be simply updated in every period (Brock and Hommes, 1997) as

$$U_{x,t}^k = \rho U_{x,t-1}^k - (E_{t-2}^k x_{t-1} - x_{t-1})^2, \tag{8}$$

where the memory parameter ρ (with $0 \leq \rho \leq 1$) determines the speed of a geometric dilution of the impact of the lags of $U_{x,t}^k$ on the current utility of the forecast performance. Here, $\rho = 0$ suggests that agents do not consider the past observations of $U_{x,t}^k$ in their updating scheme (8), while $\rho = 1$ means that they assign the same weight of 1 to all past observations. The switching mechanism is that agents can adaptively revise their expectations given the forecast performance of particular heuristics based on the multinomial logistic model:

$$\alpha_{x,t}^k = \frac{\exp(\gamma U_t^k)}{\sum_{i=1}^3 \exp(\gamma U_t^{k\{i\}})}, \tag{9}$$

where the parameter $\gamma \geq 0$ denotes the intensity of choice, a parameter crucial to the stability of the system (cf. Anufriev et al., 2018a; Gaunersdorfer et al., 2008; Hommes, 2013; Jang and Sacht, 2016, among others). The higher γ is, the more agents are willing to learn from their past forecast performance and, therefore, are keener on switching to the best-performing forecasting strategy (De Grauwe and Ji, 2020). Note also that a negative γ lacks economic sense because it would imply irrational switching towards less precise forecasting heuristics.

Although we consider the BR model by Hommes et al. (2019) as our flagship approach due to the existing microevidence for the applied heuristics, in Online Appendix C, we conduct an additional Monte Carlo exercise for various different switching models that are directly taken from the macroeconomic literature. Our focus is on the estimation of models with alternative sets of heuristics since transferring the SML technique from financial economics to macroeconomics is the main goal of our paper. We believe this additional robustness analysis markedly fosters our intentions in this regard and enables us to forestall potential criticism related to the “wilderness of bounded rationality” argument of Sims (1980).

3.3. The model under rational expectations

Under RE, the lead terms are described by the expectations of the output gap and inflation rate at time $t+1$ in Eqs. (1) and (2):

$$\tilde{E}_t^{RE} x_{t+1} = E_t x_{t+1}, \tag{10}$$

with, again, $x = \{y, \pi\}$, and where E_t denotes the statistical expectation operator conditional on information at time t. For the corresponding random error term usually to be found in Eq. (10) and denoted by $\tilde{\varepsilon}_{x,t}$, $E_t \tilde{\varepsilon}_{x,t} = 0$ holds since it is independent of the future realizations in x. This implies that agents' expectations are not systematically biased under RE.

In the Monte Carlo study and empirical application, we follow Jang and Sacht (2016) and consider the BR NKM and the baseline RE NKM in the hybrid version with a lead and lag structure. With respect to the former, $\chi = \alpha = \phi_r = 0$ holds. We distinguish between the two specifications according to a potential “persistence anomaly” (Chari et al., 2002), where the DSGE models that produce only monotonic dynamic patterns fail to empirically capture observable hump-shaped movements in output and inflation. This is indeed true for a purely forward-looking RE model, i.e., without any lag terms in the demand and supply Eqs. (1) and (2), especially in the absence of autocorrelated shocks. However, in the BR model framework, the inertia in the model variables is ensured based on the heuristics (4)–(6) being applied. Being aware of the persistence in real-world data, we therefore explicitly account for intrinsic persistence due to the assumption of consumption habits, price indexation, and interest rate smoothing in the hybrid baseline NKM under RE. This explains why the term “hybrid” is attached to the RE NKM expression because it helps to clearly distinguish between the two model specifications.

recent machine learning-based models (Hastie et al., 2009). However, these approaches resemble toolkits of macroeconomic analysts and professional forecasters, not rule-of-thumb-based behavior of ordinary economic agents microfounded by laboratory experiments with nonspecialist human subjects as considered in our study. Indeed, Haldane and Madouros (2012) emphasize the “less-is-more” principle, whereby the cost of cognition and information processing might outweigh the desired optimized response in a complex environment. Gigerenzer and Brighton (2009) empirically show that a biased mind can achieve substantially more accurate results regarding desired preferences when using heuristics. Therefore, resource-intensive forecasting strategies might be subconsciously avoided due to humans' limited cognitive abilities, although they may increase forecast accuracy in a complex environment.

3.4. Model solutions and estimation

The state-space representation of the baseline NKM is given by

$$\mathbf{AX_t} + \mathbf{BX^j_{t+1}} + \mathbf{CX_{t-1}} + \mathbf{D\Gamma_t} = \mathbf{0}, \tag{11}$$

with $\mathbf{X_t} = (y_t, \pi_t, r_t)'$, $\mathbf{X^j_{t+1}} = (\tilde{E}^j_t y_{t+1}, \tilde{E}^j_t \pi_{t+1}, \tilde{E}^j_t r_{t+1})'$, $\mathbf{X_{t-1}} = (y_{t-1}, \pi_{t-1}, r_{t-1})'$ and $\mathbf{\Gamma_t} = (\varepsilon_{y,t}, \varepsilon_{r,t}, \varepsilon_{\pi,t})'$. The corresponding general reduced-form solution of

$$\mathbf{X_t} = -\mathbf{A^{-1}}[\mathbf{BX^j_{t+1}} + \mathbf{CX_{t-1}} + \mathbf{D\Gamma_t}] \tag{12}$$

is then obtained by applying the method of undetermined coefficients. For the BR specification, the forward-looking elements in X^{BR}_{t+1} are replaced by the forecasting heuristics (4)–(6), obviously except for the expectations on the interest rate $\tilde{E}^j_t r_{t+1}$. Since the BR NKM exhibits a purely backward-looking structure due to the heuristics (4)–(6), the reduced-form solution (12) is considered directly when estimating the BR model.[5] The solution for the hybrid RE NKM is instead obtained by applying the brute force iteration method introduced by Binder and Pesaran (1999).

With respect to the BR model specification, in the following section, we describe how the SML method estimates the *structural* $(\tau, \kappa, \phi_y, \phi_\pi)$ and the *bounded rationality* $(\eta, \iota, \mu, \gamma)$ parameters. For the RE model, the parameters for habit formation, price indexation, and interest rate smoothing (χ, α, and ϕ_r) are estimated, while the BR parameters are not considered.

4. The simulated maximum likelihood approach for the NKM

This section introduces the SML estimator to macroeconometrics. The SML method is primarily known from the financial econometric literature on the estimation of univariate time-series models (Kristensen and Shin, 2012; Kukacka and Barunik, 2017; Lee and Song, 2015). For macroeconomic optimization problems, it requires subsequent modification to a multivariate version.

Let us assume a generic multivariate time-series process (z_t, x_t), $z_t : t \mapsto \mathbb{R}^l$, $l \in \mathbb{N}$; $x_t : t \mapsto \mathcal{X}_t$, $t = 1, \ldots, \infty$. Suppose that we have T realizations $\{(z_t, x_t)\}_{t=1}^T$. We further assume that the time series $\{z_t\}_{t=1}^T$ has been generated by a fully parametric model:

$$z_t = m_t(x_t, \varepsilon_t, \theta), \quad t = 1, \ldots, T, \tag{13}$$

where a model function m_t maps $\{x_t, \varepsilon_t, \theta\}$ to $\mathbb{R}^l$, $\theta \in \Theta \subseteq \mathbb{R}^n$ is an unknown parameter vector, and $\varepsilon_t \in \mathbb{R}^l$ is an i.i.d. sequence with known distribution $\mathcal{F}_\varepsilon$, which is assumed to be disconnected from t or θ. Realizations z_t directly represent empirical observables. In general, both multivariate processes (z_t, x_t) can be nonstationary, x_t is also allowed to contain other exogenous explanatory variables in addition to lagged dependent variables z_t, and the space $\mathcal{X}_t$ can be time-varying. Finally, we assume the model to have an associated conditional density $p_t(z|x;\theta)$:

$$P(z_t \in A | x_t = x) = \int_A p_t(z|x;\theta)dz, \quad t = 1, \ldots, T, \tag{14}$$

for any Borel set $A \subseteq \mathbb{R}^l$.

Let us now for intuitiveness consider the case of the heuristic switching model suggested in Section 3.2. For the given NKM specification, $z_t = \{y_t, \pi_t, r_t\}$; therefore, $l = 3$ following Eqs. (1) to (3). x_t only contains lagged dependent variables z_t, i.e., no other exogenous explanatory variables are considered in this model specification. ε_t stands for a set of l idiosyncratic shocks $\{\varepsilon_{y,t}, \varepsilon_{\pi,t}, \varepsilon_{r,t}\}$ that are i.i.d. around mean zero with variance $\sigma^2_{\{y,\pi,r\}}$. Finally, the set of estimated parameters contains all *structural* and *bounded rationality* coefficients, i.e., $\theta = \{\tau, \kappa, \phi_y, \phi_\pi, \eta, \iota, \mu, \gamma\}$ and $n = 8$. Recall that for the intrinsic persistence parameters $\chi = \alpha = \phi_r = 0$ hold, and the remaining model parameters are parameterized. In contrast, for the hybrid RE NKM, $\theta = \{\chi, \alpha, \tau, \kappa, \phi_y, \phi_\pi, \phi_r\}$, i.e., $n = 7$.

As a result of the adaptive revisions of the expectations introduced by Eq. (9), the probability $p_t(z|x;\theta)$ in Eq. (14) does not have a closed-form representation. Therefore, an exact mathematical derivation of the likelihood function of the model in Eq. (13) does not exist, and a standard estimator of θ, the maximizer of the conditional log-likelihoods

$$\tilde{\theta} = arg \underbrace{max}_{\theta \in \Theta} L_T(\theta), \tag{15}$$

where $L_T(\theta) = \sum_{t=1}^T \log p_t(z_t|x_t;\theta)$, is infeasible.[6] However, we are always able to obtain simulated observations from the model (13). The SML method presented below then allows us to numerically compute a simulated conditional density, which we use to obtain a simulated version of the MLE.

[5] In general, due to agents expectations revision process taking place in each period under BR, an analytical solution is difficult to be obtained. See Jang and Sacht (2016, their Appendix A) for a discussion.

[6] Moreover, the usual assumptions for the consistency and asymptotic normality of the MLE in stationary and ergodic models are imposed on the actual log-likelihood function $L_T(\theta)$ and the associated MLE to ensure that the actual, yet infeasible, MLE $\tilde{\theta}$ is asymptotically well-behaved.

To obtain a simulated approximation of the conditional density $p_t(z_t|x_t;\theta)$, $t = 1,\dots,T$, we first generate $N\times T, N \in \mathbb{N}$, i.i.d. draws from the l-dimensional distribution $\mathcal{F}_\varepsilon$, and $\{\varepsilon_{i,t}\}_{i=1}^N$ to compute

$$Z_{i,t}^\theta = m_t(x_t,\varepsilon_{i,t},\theta), \quad i = 1,\dots,N. \tag{16}$$

These N simulated i.i.d. random l-multiples, which are labelled $\{Z_{i,t}^\theta\}_{i=1}^N$, follow the target distribution by construction: $Z_{i,t}^\theta \sim p_t(\cdot|x_t;\theta)$. Thus, we can utilize them to estimate the conditional density $p_t(z|x;\theta)$ via a standard kernel approximation method. Let us define

$$\hat{p}_t(z_t|x_t;\theta) = \frac{1}{N}\sum_{i=1}^N K_H(Z_{i,t}^\theta - z_t), \tag{17}$$

where $K_H(\psi) = K(\psi/\sqrt{H})/\sqrt{H}$, $K:\mathbb{R}^l \mapsto \mathbb{R}$ is a generic kernel function that is a symmetric multivariate density and H is a symmetric positive-definite $l\times l$ bandwidth matrix.

Using the simulated conditional density $\hat{p}_t(z_t|x_t;\theta)$, we can derive the SML estimator of θ:

$$\hat{\theta} = arg \underbrace{max}_{\theta\in\Theta} \hat{L}_T(\theta), \tag{18}$$

where $\hat{L}_T(\theta) = \sum_{t=1}^T \log \hat{p}_t(z_t|x_t;\theta)$. We use the same set of draws from $\mathcal{F}_\varepsilon(\cdot)$, $\{\varepsilon_{i,t}\}_{i=1}^N$ for all values of θ. If $\hat{L}_T(\theta)$ is continuous and differentiable in θ, then numerical optimization is facilitated. Considering Eq. (17), if K and $\theta \mapsto m_t(x_z,\varepsilon_t,\theta)$ are $q \geq 0$ continuously differentiable, then the same holds for $\hat{L}_T(\theta)$. Under the regularity conditions on the conditional density p_t and kernel K (Kristensen and Shin, 2012, conditions A.1–4, K.1–2, pp. 80–81), $\hat{p}_t(z_t|x_t;\theta) \xrightarrow{P} p_t(z_t|x_t;\theta)$, which implies that $\hat{L}_T(\theta) \xrightarrow{P} L_T(\theta)$ as $N \longrightarrow \infty$ for a given $T \geq 1$. Thus, the SML estimator, $\hat{\theta}$, retains the same properties as the infeasible MLE, $\tilde{\theta}$, as $T, N \longrightarrow \infty$ under suitable conditions. Advantages and limitations of the SML over the alternative estimation approaches mentioned in Section 2 as well as its additional important properties are discussed in Online Appendix A.

5. Monte Carlo study

This section numerically investigates the finite-sample properties of the SML estimator in a macroeconomic framework. We conduct an extensive Monte Carlo simulation study to determine its capability to consistently recover the pseudo-true parameters in a controlled environment. We focus on issues such as the sources of estimation bias and uncertainty, potential specifications and parameterizations of the model, and the computational setup of the optimization procedure. The following two sections summarize the general settings for all numerical exercises if not explicitly stated otherwise.[7]

5.1. Model parameterization for the study

Starting with the BR NKM, we first parameterize the discount factor $\nu = 0.99$ whose value has strong support in the related empirical estimation literature. Other structural parameters (τ, κ, ϕ_y, ϕ_π), which are subject to estimation, and the standard deviations of the idiosyncratic shocks ($\sigma_y = 0.543$, $\sigma_\pi = 0.240$, $\sigma_r = 0.151$) are parameterized according to the recent results for the US data by Jang and Sacht (2021, their Table 2, EFB scenario). The idiosyncratic shocks $\varepsilon_{\{y,\pi,r\}}$ are therefore independently drawn from a normal distribution with mean zero and variance $\sigma^2_{\{y,\pi,r\}}$, which is a standard and reasonably realistic assumption for the distribution $\mathcal{F}_\varepsilon$ in Eq. (13). We also exploit the favorable theoretical properties of the Gaussian kernel (Kristensen and Shin, 2012, pg. 81) related to Eq. (17). The standard deviations of the idiosyncratic shocks $\sigma_{\{y,\pi,r\}}$ cannot be subject to estimation, as the SML method requires the distribution $\mathcal{F}_\varepsilon$ of the i.i.d. shocks to be known.

The BR parameters (η, ι, μ) follow the parameterization by Hommes et al. (2019) except that our trend-following parameter ι is defined as an average of their "weak" and "strong" trend-following parameters to avoid issues with parameter identification. The parameter for the intensity of choice is set to $\gamma = 1$, which follows the economic intuition of a low intensity of switching between forecasting heuristics since agents show a moderate willingness to learn from their past performance at the quarterly horizon. This implies that they tend to maintain their current forecasting strategy, which rules out erratic switching dynamics, i.e., sudden turnovers towards a single dominant forecasting strategy do not occur. Hence, the switching process turns out to be less intense.[8] Finally, no memory of the past forecast performance is assumed, $\rho = 0$,

[7] All computations are conducted using Julia version 1.4.0 (2020-03-21). The computational burden becomes manageable by utilizing the Distributed.jl package for parallel computing on multicore computers/servers. For optimization, the BFGS algorithm from the Optim.jl package is used with an additional LineSearches.jl (`.BackTracking`) functionality to decide the step length. BFGS is a gradient-based iterative optimization algorithm that searches for a local minimum of a differentiable function. In our specification, it minimizes the negative simulated log-likelihood function $\hat{L}_T(\theta)$. BFGS approximates the Hessian using differences in the gradient across iterations. The starting matrix is an identity matrix. The BFGS algorithm was tested against other algorithms available in the `Optim.jl` package, and it delivers the best performance for the given optimization problem. It also delivers a largely comparable performance to differential evolution algorithms from an alternative BlackBoxOptim.jl package.

[8] Our choice of the numerical value of γ closely follows the related literature. For comparison, De Grauwe (2010, 2011, 2012), Liu and Minford (2014), Deák et al. (2017), Jang and Sacht (2021) also set $\gamma = 1$, Hommes et al. (2019) calibrate this parameter to 0.4, De Grauwe and Ji (2020) set $\gamma = 2$, and

based on the results of Jang and Sacht (2016, 2021), who generally find this parameter to be insignificant for the heuristic switching model using the SMM estimation technique.

For the hybrid RE NKM, we again set the discount factor by $\nu = 0.99$. The structural parameters together with the standard deviations of the shocks follow common parameterizations based on Hommes et al. (2019) and De Grauwe and Ji (2020).

Accordingly, the values of the pseudo-true parameters that are subject to estimation are listed in the second and sixth columns of Table 1. However, the qualitative results presented are robust with respect to other realistic parameterizations of the model, e.g., based on additional results by Jang and Sacht (2021) and parameterization setups by Hommes et al. (2019) and De Grauwe and Ji (2020) to which we devote attention in Online Appendix B.

5.2. Simulation setup

We study the performance of the SML estimator under three lengths of time series generated from the model, namely, 250, 500, and 5000. This allows us to maintain an admissible time span for common quarterly macroeconomic data while studying the asymptotic tendencies of the estimator with an increasing sample size. Moreover, 1000 additional initial simulated observations are always discarded as a burn-in period. Next, to ensure the statistical validity of the results while keeping the computational burden manageable, 300 independent Monte Carlo runs are always conducted. The random seed is controlled at the level of individual runs, which allows for the replicability of the results.

In the general setup, we apply a constrained optimization of a multivariable function and jointly estimate eight parameters for the BR NKM or seven parameters for the hybrid RE NKM. The parameter space is restricted to the intervals summarized in Table 1. The constraints are based on the theoretical borders for given parameters (cf. Section 3.2) combined with a preliminary rough search based on a broader space to verify the sufficient lengths of the intervals. The random starting point for the optimization search is uniformly generated from given intervals. The precision of the kernel density approximation is set to $N = 1000$. For the estimation of the conditional density $p_t(z|x;\theta)$, we consider the multivariate Gaussian kernel and Silverman's (1986) rule of thumb to set an optimal bandwidth matrix: $\sqrt{H_{s,s}} = \big(4/[(l+2)N]\big)^{1/(l+4)}\widehat{\sigma_s}$, where $l = 3$, $s = \{1, \dots, l\}$, and $\widehat{\sigma_s}$ denotes the sample standard deviation of the elements of the sth dimension of $\{\varepsilon_i\}_{i=1}^N$ and off-diagonal terms $H_{s_1,s_2} = 0$, $s_1 \neq s_2$.

5.3. Numerical results

We primarily investigate the capability of the SML estimator to recover the pseudo-true parameters in finite samples. The focus is on the estimation precision for the BR NKM and on a discussion of the probable sources of potential estimation bias. Another dimension of the analysis is a numerical evaluation of the asymptotic tendencies to the consistency and efficiency of the estimator. A comparison with the hybrid RE NKM is also provided.

5.3.1. Estimation accuracy

Table 1 reveals very promising performance of the SML estimator. An alternative graphical depiction of the identical results is provided in Fig. 1. First, by focusing on the BR NKM in the left half of the table/figure, the values of the Taylor rule coefficients ϕ_y and ϕ_π are recovered nearly perfectly, and the 95% sample confidence intervals suggest a minimal variance of the estimator for the given parameters. The slope of the New Keynesian Phillips curve κ is also estimated very precisely. For the elasticity of substitution in consumption τ, we observe positively biased results, i.e., a considerable deviation of the numerical point estimate from the pseudo-true value for all T, that suggest a stable bias of the SML estimator for this parameter. The sampling distributions have regular symmetric shapes with skewness close to zero and kurtosis close to a normal distribution.

The bias for τ can be explained at the level of the model structure as follows. Equation (1) essentially plugs r_t (3) into the dynamic IS curve where it directly interacts with τ. This creates a composite error term $\varepsilon_{y,t} + \tau\varepsilon_{r,t}$ that introduces a correlation between the individual errors in the model. It also naturally correlates with τ. Moreover, τ thus also loads directly on the Taylor rule coefficients and indirectly on κ through the $\phi_\pi \pi_t$ term in Eq. (3). All of this leads to a problematic identification of τ that results in a bias of the estimator for this parameter. This seems to be a fundamental finding gained for the interpretation of the empirical results, where unfortunately, we can hardly predict even the direction of the bias for τ. One potential solution analyzed in Online Appendix B is to fix this parameter. It is important to emphasize, however, that in such a complicated nonlinear system as the BR NKM, similar biases are practically inevitable. The purpose of Monte Carlo situations is exactly to learn about their existence as concretely as possible through numerical simulations. Moreover, in Section 6.3, we provide an overview of existing empirical estimates of this parameter.

The BR parameters are generally more challenging to estimate, but the SML method still delivers very good performance. The sample variances for η, ι, and μ are relatively larger than for the structural parameters; however, the sampling distributions have regular and reasonably symmetric shapes with kurtosis generally greater than a normal distribution. We also

Grazzini et al. (2017) use $\gamma = 5$ when estimating the De Grauwe (2012) model. Özden (2021) and Fischer (2022) report parameter estimates for γ given by 2.1 and 78.6, respectively. Jang and Sacht (2016) employ a range $\gamma = \{0.1, 1, 10\}$ for negligible, smooth and slow, and strong switching, respectively, to estimate the De Grauwe (2011) model.

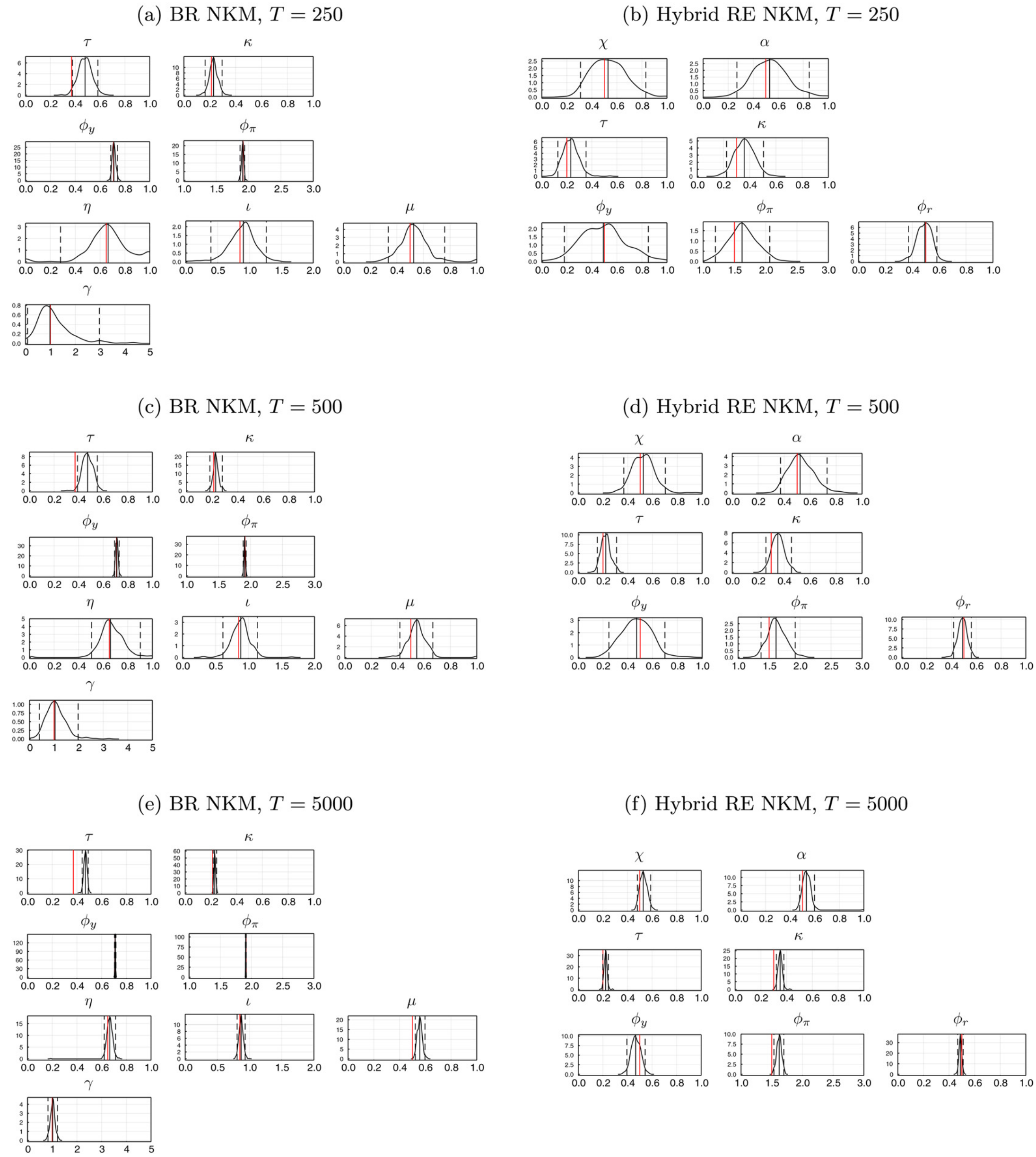

Fig. 1. Densities of the pseudo-true parameter estimates. *Note:* The bold black curves depict the kernel density estimates of the sample densities, the bold red vertical lines show the pseudo-true values, and the dashed red vertical lines depict the 95% confidence intervals of the sample estimates. Based on 300 random runs, the parameterization follows Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

do not observe any considerable bias tendencies except for a small positive bias for μ. The most important result is undoubtedly the estimation performance for the switching parameter of the intensity of choice γ. Even in simple univariate financial models, capturing the effect of this coefficient is generally difficult (Boswijk et al., 2007; Hommes, 2013; Lamperti, 2018a; Lux and Zwinkels, 2018). In addition, a handful of previous macroeconomic studies have had difficulties estimating this parameter (cf. Section 2). On the contrary, our results show that the SML estimator is well capable of estimating the pseudo-true intensity of choice γ under the multinomial logistic updating scheme (9). That is, compared to other estimation methods applied to date, the SML estimator demonstrates a considerable ability to estimate γ without any bias and without issues with statistical insignificance at the standard 5% level. It is thus capable of detecting signs of behavioral switching in the simulated model output.

Finally, in Online Appendix C, Figure 6, we report results for the application of the SML estimator to different heuristic switching models derived based on alternative sets of heuristics. Specifically, we estimate the models following the heuristic

Table 1
Results of the Monte Carlo simulations.

	Par.	BR NKM			Par.	Hybrid RE NKM		
		T=250	500	5000		T=250	500	5000
χ	–	–	–	–	*.50*	.53	.53	.53
$\langle 0, 1\rangle$						(.31-.83)	(.37-.70)	(.48-.59)
α	–	–	–	–	*.50*	.53	.52	.53
$\langle 0, 1\rangle$						(.27-.85)	(.37-.73)	(.48-.60)
τ	*.371*	.48	.47	.47	*.20*	.23	.22	.22
$\langle 0, 1\rangle$		(.38-.58)	(.39-.55)	(.44-.49)		(.13-.35)	(.15-.31)	(.20-.24)
κ	*.213*	.23	.23	.23	*.30*	.36	.35	.35
$\langle 0, 1\rangle$		(.16-.30)	(.18-.28)	(.22-.24)		(.22-.51)	(.26-.46)	(.32-.38)
ϕ_y	*.709*	.71	.71	.71	*.50*	.49	.47	.47
$\langle 0, 1\rangle$		(.69-.74)	(.69-.73)	(.70-.72)		(.18-.85)	(.25-.70)	(.39-.54)
ϕ_π	*1.914*	1.91	1.91	1.91	*1.50*	1.62	1.61	1.62
$\langle 1, 3\rangle$		(1.87-1.94)	(1.89-1.93)	(1.90-1.92)		(1.20-2.06)	(1.37-1.92)	(1.54-1.70)
ϕ_r	–	–	–	–	*.50*	.49	.49	.49
$\langle 0, 1\rangle$						(.37-.58)	(.42-.56)	(.47-.51)
η	*.65*	.67	.66	.67	–	–	–	–
$\langle 0, 1\rangle$		(.28-1)	(.51-.90)	(.62-.71)				
ι	*.85*	.90	.88	.87	–	–	–	–
$\langle 0, 2\rangle$		(.39-1.26)	(.61-1.13)	(.81-.93)				
μ	*.50*	.53	.54	.56	–	–	–	–
$\langle 0, 1\rangle$		(.34-.76)	(.42-.67)	(.52-.60)				
γ	*1.00*	.98	1.04	1.01	–	–	–	–
$\langle 0, 5\rangle$		(.08-2.98)	(.40-1.98)	(.83-1.21)				

Note: The constraints for optimization and its starting point are given in $\langle\rangle$ brackets. T denotes the length of the executed time series. The sample medians based on 300 random runs are reported, while the 95% confidence intervals of the sample estimates are reported in () parentheses. The parameterization is based on Jang and Sacht (2021), their Table 2, EFB, for the BR NKM and on Hommes et al. (2019), De Grauwe and Ji (2020) for the hybrid RE NKM. The figures are rounded to 2 or 3 decimal places.

structure by Gaunersdorfer and Hommes (2007), De Grauwe (2011), De Grauwe and Ji (2020) and with τ parameterized to avoid estimation bias. The accuracy and efficiency of the SML estimator for these alternative models is largely comparable to our main findings and thus provides a strong robustness check to our Monte Carlo simulation study. As a closing exercise, Online Appendix D, Table 4 and Figure 7, report the results of a direct numerical comparison of the estimation performance between the SML method and the SMM, which suggests the superiority of the SML estimator for the given model setup.

5.3.2. Asymptotic tendencies

Favorable asymptotic tendencies of the estimator are apparent with an increasing sample size $T = 250, 500, 5000$. We generally observe a considerable narrowing of the 95% confidence intervals of the sample estimates when T goes from 250 to 500 and further to 5000. This tendency leads to markedly stronger improvements of the estimation efficiency for the BR parameters compared to the structural parameters. The results further demonstrate significant improvements in the estimation precision for the intensity of choice γ with an increasing sample size. Moreover, although we observe a positive skew of the sampling distribution of the estimator for γ for $T = 250$, this asymmetry decreases for $T = 500$ and disappears completely for $T = 5000$. The same normalization of the shape of the sapling distribution can be observed at the level of kurtosis. For $T = 250, 500$, the excess kurtosis is slightly positive for the structural parameters and rather large for the BR parameters, whereas for $T = 5000$, such a nonnormal shape disappears with the excess kurtosis for all parameters approaching zero. In contrast, the bias of the SML estimator for parameters τ and μ appears to be stable regardless of the sample size. A very subtle tendency towards bias reduction can be observed for a small bias for ι.

5.3.3. Shape of the log-likelihood function

A set of regularity conditions, A.1–4, that impose restrictions on the model and the conditional density, are defined in Kristensen and Shin (2012) such that the estimated conditional density $\hat{p}$ converges sufficiently fast to the true conditional density p. Therefore, the asymptotic equivalence of the estimated $\hat{\theta}$ and the true θ parameter vectors are assured. Kristensen and Shin (2012) assert that these assumptions are "quite weak and are satisfied by many models"; nonetheless, the analytical intractability of the analyzed heuristic switching macroeconomic model does not allow us to mathematically verify these conditions. Therefore, we take advantage of the computational approach and verify the smoothness condition, identification of parameters, and existence of a unique optimum by assessing the simulated log-likelihood functions via graphical tools.

The left half of Fig. A.1 in Appendix A shows the shape of the simulated log-likelihood function for the BR NKM. As we are unable to graphically depict an 8-dimensional object, the individual curves show the transversal profiles of the simulated log-likelihood function in the planes of given parameters. Other parameters are held fixed at their parameterized values (Table 1). A smooth surface of the average simulated log-likelihood functions resulting from the use of a Gaussian kernel based on $N = 1000$ approximation points is clearly observable for all structural parameters and the intensity of choice γ over the entire parameter space. Moreover, the unique maxima are well detectable for the structural parameters with the strongest curvature in the direction of the parameter ϕ_π, which naturally corresponds to the numerical results reported in Table 1 and Fig. 1. For the remaining BR parameters except for γ, one can observe considerable differences and irregularities between some of the independent realizations of the simulated log-likelihood function. Together with a rather flat surface of the average likelihood function, these differences are likely the cause of higher sample variances for η, ι, and μ. This observation explains our findings regarding the more challenging estimation of these parameters. The bias for τ is also obvious from the shape of the simulated log-likelihood. When increasing the sample size from $T = 500$ (panel c) to $T = 5000$ (panel e), we obtain distinctively sharper shapes with unique maxima observed in the directions of all estimated parameters, suggesting their proper identification. We also gain considerably more regular behavior of the log-likelihood for all independent runs. Accordingly, we can generally assume that the regularity conditions are satisfied for the BR NKM.

5.3.4. Comparison with the RE model

The results for the hybrid RE NKM are also summarized in Table 1 and the right half of Fig. 1. The estimation performance is generally worse for the hybrid RE NKM than for the BR NKM, while the number of estimated parameters is one smaller. This result might appear counterintuitive at first, since the BR NKM is expected to generate more complex dynamics than the RE NKM. One reason for this finding might be the qualitatively different behavior of the output time series of the realizations in the RE case. In contrast, in the model under BR, the output displays considerable stability and path dependency that emerges from the incorporation of behavioral heuristics, and the fluctuations of the output time series of the hybrid RE NKM are more intense, regardless of the impact of the intrinsic persistence parameters χ, α and ϕ_r. Exogenous random shocks rather than (cross-)autocorrelation structures thus seem to be a driver for the hybrid RE NKM dynamics, which would naturally complicate the proper detection of the pseudo-true parameter values for any estimation method. The inability of the SML estimator to approximate the likelihood function accurately enough for more volatile time series is widely studied in Kukacka and Barunik (2017). In such cases, the SML criterion function displays a flatter surface that hinders the optimization algorithm from reaching the global optimum in a multidimensional parameter space. This can generally be observed in the right half of Fig. A.1 in Appendix A, especially for parameters κ, ϕ_y, and ϕ_π in which the individual courses of the depicted shapes are more dispersed.

Another related issue regards small biases of the estimator that are mostly visible for parameters κ and ϕ_π. Moreover, the bias does not tend to disappear as the sample size increases, which suggests unfavorable asymptotic tendencies. In terms of efficiency, the 95% sample estimate intervals are rather large for the intrinsic persistence parameters χ and α and for the Taylor rule coefficients ϕ_π and ϕ_y. However, for ϕ_π and ϕ_y the estimation performance is incomparable to that of the BR NKM because the interest rate smoothing parameter ϕ_r takes over their impact. The relative improvement in efficiency as the sample size increases seems comparable.

6. Empirical applications

This section examines the empirical performance of the SML in a multivariate macroeconomic setup. We are also interested in evaluating and reporting the parameter estimates used for activities in policy advisory. Both the BR NKM and its hybrid RE NKM counterpart are considered since a purely forward-looking RE NKM without lag terms does not capture the empirically observed inertia in inflation and output due to the aforementioned “persistence anomaly” (Chari et al., 2002). Since the data for the US economy exhibit a high degree of persistence, it is worthwhile to investigate different model specifications where inertia is technically incorporated either by heuristics (BR NKM) or by the assumption regarding habit formation and price indexation (hybrid RE NKM).

6.1. Estimation setup

The algorithm for empirical estimation generally adopts the setup of the simulation study in Sections 5.1 and 5.2 with two minor alterations. First, compared to the parameter space for the numerical study, we restrict the interval for the intensity of choice γ by 10 from above based on a preliminary rough search. We further extend the interval for the Taylor rule coefficient ϕ_π to $\langle 0, 3\rangle$, which supports an optimization search. Second, we increase the precision of the kernel density approximation to $N = 2000$. For comparison exercises, we consider three additional parameter sets where we parameterize the structural parameters τ and/or κ as summarized in Table 2.

6.2. Data

We obtain the US quarterly data from the website of the Federal Reserve Bank of St. Louis.[9] Inflation is measured using the seasonally adjusted consumer price index (for all urban consumers & items). Output is obtained from seasonally adjusted real GDP based on billions of chained 2012 dollars. A standard smoothing parameter of $\lambda = 1600$ is used to estimate the

[9] Available at fred.stlouisfed.org [accessed 9 September 2019].

Table 2
Results of the empirical study for US data: BR vs. hybrid RE NKM.

	BR NKM			Hybrid RE NKM		
	All	B	C	All	D	All via MLE
χ	–	–	–	.99	.74	1.00
$\langle 0, 1\rangle$				(.81-1.00)	(.67-.999)	(.88-1.00)
α	–	–	–	.77	.999	.81
$\langle 0, 1\rangle$				(.70-.83)	(.90-1.00)	(.75-.87)
τ	.00	*.371*	*.20*	.05	.02	.04
$\langle 0, 1\rangle$	(.00-.00)			(.02-.08)	(.004-.04)	(.03-.05)
κ	.09	*.213*	*.05*	.00	*.03*	.00
$\langle 0, 1\rangle$	(.007-.17)			(.00-.00)		(.00-.004)
ϕ_y	.12	.05	.04	.78	.69	.79
$\langle 0, 1\rangle$	(.10-.14)	(.03-.07)	(.01-.06)	(.60-.98)	(.45-.93)	(.49-1.09)
ϕ_π	1.26	1.23	1.27	1.28	1.32	1.51
$\langle 0, 3\rangle$	(1.14-1.31)	(1.19-1.29)	(1.22-1.33)	(1.03-1.54)	(.26-1.58)	(1.23-1.79)
ϕ_r	–	–	–	.91	.90	.93
$\langle 0, 1\rangle$				(.88-.93)	(.87-.93)	(.90-.95)
η	.19	.21	.20	–	–	–
$\langle 0, 1\rangle$	(.04-.27)	(.09-.25)	(.09-.26)			
ι	.00	.00	.00	–	–	–
$\langle 0, 1\rangle$	(.00-.002)	(.00-.02)	(.00-.05)			
μ	.31	.38	.44	–	–	–
$\langle 0, 1\rangle$	(.07-.67)	(.15-.62)	(.19-.65)			
γ	1.40	1.49	1.39	–	–	–
$\langle 0, 10\rangle$	(.50-7.26)	(.79-6.22)	(.78-6.35)			

Note: The constraints for optimization and its starting point are given in ⟨⟩ brackets. The sample medians based on 300 random runs are reported, while the 95% confidence intervals of the sample estimates are reported in () parentheses. The parameterization is based on Jang and Sacht (2021), their Table 2, EFB (for B), the hybrid RE (for D), and De Grauwe and Ji (2020) for C. The fixed parameters are marked in italics. The figures are rounded to 2 or 3 decimal places.

trend of the observed data from the one-sided Hodrick-Prescott filter (Stock and Watson, 1999) for output. The effective federal funds rate is used to measure the short-term nominal interest rate in the US. The sample covers the period from 1954:Q3 to 2019:Q2 with 260 observations in total.

6.3. Empirical results

The parameter estimates for both model specifications are shown in Table 2. We first focus on the BR NKM for which all BR parameters are statistically significant (except for the trend-following parameter ι; see below). Note that the column entitled “All” reports the results for which none of the parameters are parameterized to a predefined fixed value. Under this scenario, the adaptation parameter η exhibits a median value of 0.19, i.e., approximately 20% of the past realization in the output gap, and the inflation rate is considered in case the adaptive heuristic (4) is applied. Therefore, a naïve expectation formation process, with η being close or equal to unity, can be ruled out. The estimate of the anchoring parameter μ given by 0.31 reveals a weaker importance of the corresponding anchor in the heuristic (6) than suggested by Hommes et al. (2019).

A novelty of our empirical investigation is that we are reliably able to identify the intensity of choice parameter γ in a macroeconomic model setup. We obtain a clearly statistically significant estimate of 1.40. This implies that the agents have the incentive to revise their expectation formation process in every period. According to De Grauwe and Ji (2020), γ can be interpreted as “expressing a willingness to learn from past performance”. Based on our estimate, this degree of willingness turns out to be rather moderate. As a result, the switching from one forecasting heuristic to the other becomes less erratic considering the quarterly frequency of the data.[10]

We now briefly discuss the estimate for ι, which denotes the extrapolation parameter in the trend-following heuristic (5). It measures how strongly the expectation formation based on this specific forecasting rule considers the change in the direction of the output gap or the inflation rate up to the second lag. The observation that ι turns out to be insignificant is unsurprising. The reason for this is twofold. First, technically speaking, one can note that the corresponding term $(x_{t-1} - x_{t-2})$ can be found in both heuristics for trend-following and learning anchoring and adjustment, i.e., (5) and (6), respectively. The difference lies in the weighting scheme of ι and unity. An insignificant estimate for ι therefore indicates the redundancy of an additional but identical term as part of both forecasting rules. Second, note that the following economic interpretation holds with respect to heuristic (5) in isolation. By acknowledging the extrapolation effect, it gives rise

[10] Note that the results regarding the intensity of choice γ can be compared to those presented for similar representations of the BR NKM obtained via the SMM in Jang and Sacht (2016, 2021), where a Taylor rule with interest rate smoothing is assumed. The intensity of choice parameter is fixed in their study, which means that as a cross-check, the case of $\gamma = \{0.1, 1\}$ for the US should be considered since we find a median value of γ close to unity.

to strong oscillatory dynamics for a high value of ι (Anufriev and Hommes, 2012). This resembles a destabilized economy with high degrees of fluctuation in the macroeconomic variables. However, the trend-following heuristic hardly survives if the central bank reacts strongly to disturbances in output and inflation (Hommes, 2021). This is, in fact, suggested by our estimates for ϕ_y and ϕ_π. Instead, the agents' expectation formation is dominated by the other two heuristics, namely, *ADA* (4) and *LAA* (6). Our empirical results thus support the experimental findings by (Hommes et al., 2019, Figure 11) who also find that these two heuristics strongly dominate various experimental setups. According to Assenza et al. (2013), as the central bank successfully stabilizes the economy over time, changes in $(x_{t-1} - x_{t-2})$ flatten out. Such negative-feedback policies of the central bank prevent coordination on the trend-extrapolating behavior and the survival of trend-following strategies (Assenza et al., 2021). This makes the term in the *TR* heuristic (5) most likely obsolete when it comes to expectation formation, which is then reflected by ι not deviating significantly from zero.

A discussion of the estimates for the structural parameters follows. We focus again on the "All" scenario first. Although the estimates of the monetary policy parameters indicate a strong emphasis on inflation over output stabilization with median values of $\phi_y = 0.12$ and $\phi_\pi = 1.26$, the remaining structural parameters τ and κ turn out to be (close to) statistically insignificant. This observation regarding the slopes of the dynamic IS Eq. (1) and the New Keynesian Phillips curve (2) holds under the inspection of the corresponding 95% confidence intervals. At least for τ, the result ties into our discussion in Section 5.3.1 concerning a weak identification and the estimation bias for this parameter. It follows that the demand and supply curves seem to be decomposed from the remaining system of equations and that inherited persistence, in terms of cross-volatility within the inflation-output gap nexus, is largely absent.

Corresponding parameter estimates close to zero or even being insignificant are, however, not uncommon regarding the estimation of the NKM. Moons et al. (2007), for example, provide an overview of the results stemming from various studies using different techniques when estimating a RE NKM for the Euro Area. The authors report that parameter estimates for τ vary from 0.03 to 0.08. Franke et al. (2015) report values for τ obtained from applying the Bayesian technique, MLE and SMM between 0.048 and 0.113 for the Great Inflation and 0.017 and 0.275 for the Great Moderation period, respectively, in the US. In the context of the BR NKM, Jang and Sacht (2016, 2021) estimate the baseline NKM via SMM. In their 2016 paper, they present parameter estimates in the range of 0.179 and 0.737 based on Euro Area data and considering different parameterized values for γ. Estimates between 0.128 and 0.371 based on US data are reported in their 2021 paper for the same model with various combinations of heuristics. The majority of these particular parameter estimates in Jang and Sacht (2021) turn out to be insignificant.

As output and inflation dynamics seem to be characterized only by the parameterized shocks and heuristics, this raises the issue of a potential model misspecification. Although the BR NKM is by definition a perfectly specified model in the Monte Carlo environment, it struggles when dealing with an empirical data set that exhibits structural breaks in inflation persistence and volatility due to a shift in the monetary policy regime during the 1980s. Such structural breaks are, however, a widely known phenomenon for the US economy in the second half of the 20th century, where in regards to inflation, a switch to low persistence and volatility regimes can be observed for the early 1990s as discussed in a multitude of studies (see, e.g., Check and Piger, 2021; Eo, 2016, among others).

An additional exercise reveals that the estimates of all additional parameters turn out to be robust with respect to different parameterizations of τ and κ. Therefore, we report two additional sets of estimates denoted by "B" and "C" in the third and fourth columns of Table 2, respectively. For "B", both critical parameters are parameterized according to Jang and Sacht (2021, their Table 2, EFB scenario). For "C", we apply a plain vanilla parameterization following De Grauwe and Ji (2020, pg. 8). We choose both parameterizations since these are directly linked to the empirically examined BR NKMs similar to that considered in this paper.

6.3.1. Hybrid RE NKM

We now shed light on the empirical results linked to the hybrid RE NKM. We consider two different sets. In the set "All", none of the parameters are parameterized. For "D", we parameterize κ, which happens to be statistically insignificant under the "All" set. We choose a standard value of 0.03 taken from the literature on RE NKMs in general. With κ being parameterized, τ now becomes nearly insignificant as seen in the "D" column of Table 2. This is, once again, an indication of the distorted cross-correlation between output and inflation. Potential explanations include not only a misspecification of the theoretical model but also a weak identification of κ in general. This might be caused by the existence of structural breaks in the underlying time series for the US as discussed above and/or by the choice of the output gap as the driving force of inflation.[11] However, this seems to be a puzzling observation because, as in the BR NKM case, the estimation method performs well in simulations. We conclude that a further exploration of this issue is needed in future research.

We now offer a brief economic interpretation of the estimates for the hybrid RE NKM. The median values for χ, α, and ϕ_r under the set "All" come close to their upper bounds, which are given by unity in all cases. This reflects a high degree of persistence in the underlying empirical time series. It must then be mimicked by a high degree of hybridity in

[11] Speaking of a misspecification of the New Keynesian Phillips curve, the debate on the appropriate choice regarding the driving force of inflation and potential insignificant parameter estimates for κ is not new (see, e.g., Schorfheide (2008), Kohlscheen and Moessner (2022), among others). For example, Nason and Smith (2008, pg. 389) argue that "real unit labor costs are much better at statistically explaining inflation than are a plethora of output gap measures". For discussion on testing for poorly identified parameters in a single-equation estimation of the New Keynesian Phillips curve, we refer to Ahrens and Sacht (2014).

the theoretical model, i.e., high values of the parameters for habit formation (χ), price indexation (α), and interest rate smoothing (ϕ_r). The result of $\phi_\pi = 1.28$ reveals a value that is virtually identical to that obtained in the BR NKM estimates. With respect to ϕ_y, it attains a comparatively high value of 0.78, which also implies a strong focus on output stabilization.

For completeness, we exploit the linearity of the hybrid RE NKM and estimate the model via the maximum likelihood technique. The classical MLE based on the true likelihood is derived in Appendix B. We consider the "All via MLE" set where no parameters are fixed. The results are reported in the last column of Table 2. In a direct comparison to the estimates obtained via the SML method, these are hardly distinguishable from the estimates obtained through the MLE. The only exception are the estimates for ϕ_π. In addition, for the MLE the width of the 95% confidence intervals is narrower than in the "D" column but is similar to the "All" column. Overall, it can be concluded that the SML approach approximates the MLE method very well.

6.3.2. Robustness check: Great Inflation vs. Great Moderation

Finally, we empirically elaborate on the "structural break" argument and conduct a robustness check regarding two different monetary policy regimes. With a focus on a structural break in inflation rate persistence and volatility, we split the data into two subsamples: namely the so-called "PG" (in reference to the Pre-Greenspan era where at its end, Paul Volcker acted as the Fed's chairman) period from 1954 to 1987 with high inflation rates and volatility and the "G+" (in reference to Alan Greenspan and his successors) period from 1988 to 2019. Our motivation to do so is grounded on the discussion in the literature where a reduction in inflation persistence (Check and Piger, 2021) and a switch to a low inflation volatility regime (Eo, 2016) are reported for the early 1990s. The split resembles essentially dividing the full sample into the monetary policy regimes of the Great Inflation and Great Moderation periods, respectively. The other estimation setup completely follows the main estimation exercise above reported in the "All" columns in Table 2 for both the BR and the RE NKM.

Most important, the results remain robust regarding the statistical insignificance of the intertemporal elasticity of substitution τ and the slope parameter κ in the BR and the RE model, respectively. Thus, these results are not improved with a specific choice of a subsample of data, which might be expected with fewer observations. Moreover, the confidence intervals of the sample estimates generally expand because we now provide the estimation procedure with only approximately half the data points.

Interestingly, on the other hand, we can observe considerable shifts in the estimates for the intensity of choice γ and the Taylor rule coefficients ϕ_y and ϕ_π in the BR NKM. For the "PG" period, $\widehat{\gamma} = 0.76$, $\widehat{\phi_y} = 0.13$, and $\widehat{\phi_\pi} = 0.97$; while for the "G+" period, $\widehat{\gamma} = 3.39$, $\widehat{\phi_y} = 0.04$, and $\widehat{\phi_\pi} = 1.84$ are obtained. Regarding $\widehat{\gamma}$, this reflects a lower willingness and ability of economic agents to learn from their past performance during the uncertain era of high inflation volatility, i.e., in the "PG" period. This is not entirely surprising given the observation that the central bank reacted less aggressively to inflation rate fluctuations, which is reflected by the low value for $\widehat{\phi_\pi}$ and, therefore, violates the "Taylor principle". It then could be argued that a high degree of inflation rate volatility prevents intense switching movements. Agents instead tend to maintain their previously chosen forecasting heuristic because the one-period ahead inflation rate can hardly be predicted. On the contrary, in the following low inflation volatility regime, i.e., during the "G+" period, the willingness and ability of agents to learn the typical behavior of the economy are more pronounced. This is reflected by a higher value of $\widehat{\gamma}$, which seems to be linked to the central bank's strong focus on inflation stabilization for a value of $\widehat{\phi_\pi}$ close to two.

In other words, in a low-volatility economic environment, agents perform better in terms of their forecasting abilities, which is reflected by a higher value of $\widehat{\gamma}$ relative to what is obtained for the "PG" period. This leads to the observation of a more intense switching process. Our results are closely linked to the discussion on the role of the weight on inflation stabilization in the Taylor rule (3) during different monetary policy regimes in the macroeconomic literature concerning the RE NKM. For example, Lubik and Schorfheide (2004) show that US monetary policy during the 'PG" period is consistent with indeterminacy due to a passively acting central bank regarding inflation instability, while the opposite is true for the "G+" period. An earlier study by Clarida et al. (2000) also reports values of $\widehat{\phi_\pi}$ of approximately one and two for the two subsamples, respectively.

Finally, shifts in the estimated values of other behavioral parameters reveal a similar story. For the "PG" period, we have the adaptation parameter $\widehat{\eta} = 0.33$ and the anchoring parameter $\widehat{\mu} = 0.38$; while for the "G+" period, $\widehat{\eta} = 0.07$ and $\widehat{\mu} = 0.59$ are obtained. These results reveal a much higher need for periodic adaptation towards the most recent realizations of the key economic variables and weaker importance of the corresponding long-term anchors during the "PG" era of high inflation volatility compared to the stable "G+" period.

7. Conclusion

In the absence of the RE paradigm, a growing number of studies on DSGE models address the importance of boundedly rational expectation formation. This paper examines a baseline NKM with heterogeneous agents who adopt simple heuristics in forecasting future movements of output and inflation. In this heuristic switching model by Hommes et al. (2019), different groups of agents form expectations according to an adaptive, a trend-following, or a learning anchoring and adjustment scheme. The corresponding nonlinear specification of the model is for the first time estimated via the SML method (Kristensen and Shin, 2012). This approach relaxes some restrictive theoretical assumptions required by competing estimation methods and further reduces the discretionary choices necessary for their practical implementation. We modify the univariate version of the SML applied in financial econometrics and transfer it to multivariate macroeconomic optimization. The results are compared to the findings obtained for a hybrid NKM with a lead and lag structure under RE.

In a Monte Carlo simulation study, we investigate the finite-sample properties of the SML estimator in a multivariate framework. An important advantage lies in its ability to estimate the intensity of choice that governs the multinomial logistic switching process from one group to another. In previous related empirical studies, this parameter had to be parameterized or was estimated as potentially statistically insignificant or uninformative of model behavior. On the contrary, our results show that the SML can estimate the switching parameter with surprising precision and with no small-sample bias. Moreover, all three additional behavioral parameters and most of the structural parameters are also estimated very precisely. Accordingly, our numerical analysis confirms a strong capability of the SML method to estimate the heuristic switching model parameters.

In an empirical application, we estimate the model in its BR version and its hybrid counterpart under RE. Based on data on the US economy covering the period from 1954:Q3 to 2019:Q2, the majority of parameters in both models are estimated to be statistically significant. This includes most of the behavioral parameters to be found within the set of forecasting heuristics. A novelty of our analysis is that we are reliably able to identify the parameter for the intensity of choice in a macroeconomic model. Clearly statistically significant estimates between approximately 1.4 and 1.5 reveal a rather steady turnover from one forecasting heuristic to the other over time, representing a moderate willingness of economic agents to learn from their past performance. The estimation results for the model under RE are also confirmed by applying the classical MLE. However, both models partially fail to capture the cross-correlation structure within the output-inflation nexus, as the corresponding parameters turn out to be (close to) insignificant. This raises the issue of a potential model misspecification under bounded and perfect rationality.

We address this issue further by conducting an empirical robustness analysis where we split the entire sample into two subsamples that account for a high and low inflation volatility regime. The results imply that agents are reluctant to switch to another forecast heuristic in an economically uncertain environment, i.e., when a highly volatile output gap and inflation dynamics are observed. On the contrary, the degree of switching turns out to be higher in a low volatility situation where the development in the economic variables can be better predicted. This has important implications for conducting monetary policy because central banks seek to monitor inflation expectations when formulating their response to current economic disturbances. Thus, future research attempts should also explicitly consider interest rule specifications under BR, which account for expected future realizations of the output gap and inflation rate as already done in the literature on the RE NKM.

We conclude that this study considerably expands our understanding of heuristic switching models with heterogeneous agents used in macroeconomic research. Specifically, concerning the intensity of choice parameter, our empirical results are fruitful for the future parameterization of behavioral NKMs used for policy analysis. Future work in this area should focus on the estimation of more complex large-scale macroeconomic models.

Data and code availability statement

The empirical dataset that supports the findings of Section 6 can be found on GitHub at the following address: github.com/jirikukacka/Kukacka_Sacht_2022 [updated 2022-12-09]. Codes are available from the authors upon reasonable request.

CRediT authorship contribution statement

Jiri Kukacka: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration, Funding acquisition. **Stephen Sacht:** Conceptualization, Methodology, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Supervision, Project administration.

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Appendix A. Profiles of the simulated log-likelihood function

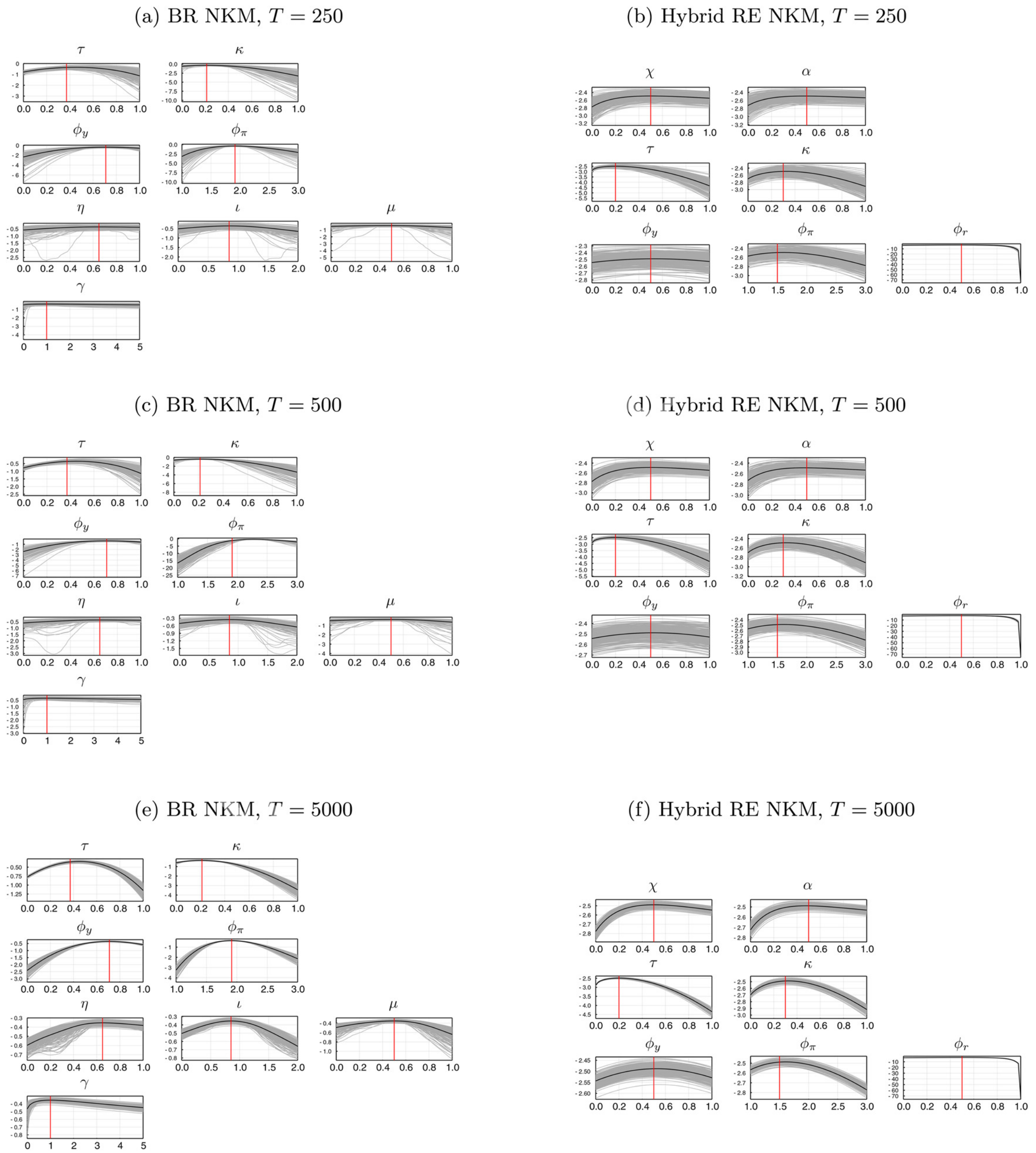

Fig. A.1. Profiles of the simulated log-likelihood function. *Note:* The bold red vertical lines show the pseudo-true values, and the bold black curves depict the average log-likelihood. Based on 300 random runs, the parameterization follows Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Appendix B. MLE for the hybrid RE NKM

This section derives the classical MLE based on the true likelihood for the hybrid RE NKM. We exploit the linearity of the RE specification of the model and follow the MLE derivation in Hamilton (1994), Lindé (2005), Jang (2012), Franke et al. (2015).[12] We also estimate the hybrid RE NKM via MLE as an additional empirical exercise, which is reported

[12] We thank Tae-Seok Jang for sharing the Matlab code with us, which is needed to apply the MLE, and his related consultations on the MLE derivation. This code was written for use in Franke et al. (2015).

in the last column of Table 2. We compare the results with those based on the SML estimator in Section 6.3 and conclude that the SML approach approximates the MLE method very well.

Based on the state space notation in Sections 3.4 and 4, the law of motion follows:

$$\boldsymbol{X_t} = \boldsymbol{\Omega X_{t-1}} + \boldsymbol{\Phi \Gamma_t}, \quad \boldsymbol{\Gamma} \sim i.i.d.\ \mathcal{N}(0, \boldsymbol{\Sigma_\Gamma}), \tag{B.1}$$

where $\boldsymbol{\Omega}$ and $\boldsymbol{\Phi}$ are the resulting RE-based projection matrices that can be obtained using a typical iterative method, and $\boldsymbol{\Sigma_\Gamma} = \mathbf{I}(\sigma_y^2, \sigma_\pi^2, \sigma_r^2)'$, where $\mathbf{I}$ is a 3×3 identity matrix. The conditional probability for the vector of the state variables $\mathbf{X_t}$ is then

$$\mathbf{X_t}|\mathbf{X_{t-1}} \sim \mathcal{N}(\boldsymbol{\Omega}\mathbf{X_{t-1}}, \boldsymbol{\Phi \Sigma_\Gamma \Phi}'). \tag{B.2}$$

The likelihood function reads as

$$L_T(\theta) = -\frac{3T}{2} ln(2\pi) - \frac{T}{2} ln|\boldsymbol{\Phi \Sigma_\Gamma \Phi}'| - \frac{1}{2} \sum_{t=2}^{T} \boldsymbol{\Gamma_t'} (\boldsymbol{\Phi \Sigma_\Gamma \Phi}')^{-1} \boldsymbol{\Gamma_t} \tag{B.3}$$

and the standard maximizer of the conditional log-likelihoods is already displayed in Eq. (15). Finally, the asymptotic properties of the MLE $\tilde{\theta}$ that allow for the estimation of the standard errors of the estimated coefficients lead to

$$\sqrt{T}(\tilde{\theta} - \theta) \sim \mathcal{N}\left(0, \left(\frac{\mathbf{H}}{T}\right)^{-1}\right), \tag{B.4}$$

where the Hessian information matrix is given by $\mathbf{H} = E\left[\frac{\partial^2 L(\theta)}{\partial\theta\partial\theta'}\right]$.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jedc.2022.104585.

References

Ahrens, S., Sacht, S., 2014. Estimating a high-frequency New-Keynesian Phillips curve. Empir. Econ. 46, 607–628.
Altissimo, F., Mele, A., 2009. Simulated non-parametric estimation of dynamic models. Rev. Econ. Stud. 76 (2), 413–450.
Andersen, T.G., Benzoni, L., Lund, J., 2002. An empirical investigation of continuous-time equity return models. J. Finance 57 (3), 1239–1284.
Anufriev, M., Chernulich, A., Tuinstra, J., 2018. A laboratory experiment on the heuristic switching model. J. Econ. Dyn. Control 91, 21–42.
Anufriev, M., Hommes, C., 2012. Evolutionary selection of individual expectations and aggregate outcomes in asset pricing experiments. Am. Econ. J. Microecon. 4 (4), 35–64.
Anufriev, M., Hommes, C., Makarewicz, T., 2018. Simple forecasting heuristics that make us smart: evidence from different market experiments. J. Eur. Econ. Assoc. 17 (5), 1538–1584.
Assenza, T., Bao, T., Hommes, C., Massaro, D., 2014. Experiments on expectations in macroeconomics and finance. In: Experiments in Macroeconomics, pp. 11–70. Chapter 2, Emerald
Assenza, T., Heemeijer, P., Hommes, C., Massaro, D., 2013. Individual Expectations and Aggregate Macro Behavior. Tinbergen Institute Discussion Papers 13-016/II. Tinbergen Institute.
Assenza, T., Heemeijer, P., Hommes, C., Massaro, D., 2021. Managing self-organization of expectations through monetary policy: a macro experiment. J. Monet. Econ. 117, 170–186.
Barde, S., 2017. A practical, accurate, information criterion for Nth order Markov processes. Comput. Econ. 50 (2), 281–324.
Barde, S., 2020. Macroeconomic simulation comparison with a multivariate extension of the Markov information criterion. J. Econ. Dyn. Control 111, 103795.
Barde, S., 2022. Bayesian Estimation of Large-Scale Simulation Models with Gaussian Process Regression Surrogates. Technical report. School of Economics, University of Kent.
Barde, S., van der Hoog, S., 2017. An Empirical Validation Protocol for Large-Scale Agent-Based Models. Technical report, Bielefeld Working Papers in Economics and Management No. 04-2017.
Binder, M., Pesaran, M.H., 1999. Multivariate Rational Expectations Models and Macroeconometric Modeling: A Review and Some New Results. John Wiley & Sons, Ltd, pp. 111–155. Chapter 3
Boswijk, H.P., Hommes, C.H., Manzan, S., 2007. Behavioral heterogeneity in stock prices. J. Econ. Dyn. Control 31 (2), 1938–1970.
Branch, W.A., McGough, B., 2018. Chapter 1: heterogeneous expectations and micro-foundations in macroeconomics. In: Hommes, C., LeBaron, B. (Eds.), Handbook of Computational Economics, Volume 4 of Handbook of Computational Economics. Elsevier, pp. 3–62.
Brock, W.A., Hommes, C.H., 1997. A rational route to randomness. Econometrica 65 (5), 1059–1095.
Brock, W.A., Hommes, C.H., 1998. Heterogeneous beliefs and routes to chaos in a simple asset pricing model. J. Econ. Dyn. Control 22, 1235–1274.
Chari, V.V., Kehoe, P.J., McGrattan, E.R., 2002. Can sticky price models generate volatile and persistent real exchange rates? Rev. Econ. Stud. 69 (3), 533–563.
Check, A., Piger, J., 2021. Structural breaks in U.S. macroeconomic time series: A Bayesian model averaging approach. J. Money Credit Banking 53 (8), 1999–2036.
Chung, H., Xiao, W., 2013. Cognitive Consistency, Signal Extraction, and Macroeconomic Persistence. Technical report. SUNY Binghamton.
Clarida, R., Galí, J., Gertler, M., 2000. Monetary policy rules and macroeconomic stability: evidence and some theory. Q. J. Econ. 115 (1), 147.
Cornea-Madeira, A., Hommes, C., Massaro, D., 2019. Behavioral heterogeneity in U.S. inflation dynamics. J. Bus. Econ. Stat. 37 (2), 288–300.
Cox, J.C., Ingersoll, J., Jonathan, E., Ross, S.A., 1985. A theory of the term structure of interest rates. Econometrica 53 (2), 385–407.
Day, R.H., Pingle, M., Franz, R., Singh, H., Gerber, J., 1991. Economizing economizing. In: Handbook of Behaviorial Economics. Greenwich: JAI Press, pp. 509–522.
De Grauwe, P., 2010. Top-down versus bottom-up macroeconomics. CESifo Econ Stud 56 (4), 465–497.
De Grauwe, P., 2011. Animal spirits and monetary policy. Econ. Theory 47 (2), 423–457.
De Grauwe, P., 2012. Booms and busts in economic activity: a behavioral explanation. J. Econ. Behav. Organ. 83 (3), 484–501.
De Grauwe, P., Ji, Y., 2019. Behavioural Macroeconomics: Theory and Policy. Oxford University Press.
De Grauwe, P., Ji, Y., 2020. Structural reforms, animal spirits, and monetary policies. Eur. Econ. Rev. 124, 103395.
Deák, S., Levine, P., Pearlman, J., Yang, B., 2017. Internal Rationality, Learning and Imperfect Information. Technical report, University of Surrey.
Del Negro, M., Schorfheide, F., Smets, F., Wouters, R., 2007. On the fit of New Keynesian models. J. Bus. Econ. Stat. 25 (2), 123–143.

Dieci, R., He, X.Z., 2018. Heterogeneous agent models in finance. In: Hommes, C., LeBaron, B. (Eds.), Handbook of Computational Economics, Volume 4 of Handbook of Computational Economics. Elsevier, pp. 257–328. Chapter 5
Dilaver, O., Jump, R.C., Levine, P., 2018. Agent-based macroeconomics and dynamic stochastic general equilibrium models: where do we go from here? J. Econ.. Surv. 32 (4), 1134–1159.
Dyer, J., Cannon, P., Farmer, J.D., Schmon, S., 2022. Black-box Bayesian Inference for Economic Agent-Based Models. Technical report.
Eo, Y., 2016. Structural changes in inflation dynamics: multiple breaks at different dates for different parameters. Stud. Nonlinear Dyn. Econom. 20 (3), 211–231.
Evans, G.W., Honkapohja, S., 1996. Least squares learning with heterogeneous expectations. Econ. Lett. 53 (2), 197–201.
Evans, G.W., Honkapohja, S., 2001. Learning and Expectations in Macroeconomics. Princeton University Press.
Fagiolo, G., Guerini, M., Lamperti, F., Moneta, A., Roventini, A., 2019. Validation of agent-based models in economics and finance. In: Beisbart, C., Saam, N.J. (Eds.), Computer Simulation Validation: Fundamental Concepts, Methodological Frameworks, and Philosophical Perspectives. Springer International Publishing, pp. 763–787.
Fischer, J.J., 2022. De-Anchored Inflation Expectations and Monetary policy. Technical report. European University Institute.
Franke, R., Jang, T.-S., Sacht, S., 2015. Moment matching versus Bayesian estimation: backward-looking behaviour in a New-Keynesian baseline model. North Am. J. Econ. Finance 31, 126–154.
Franke, R., Westerhoff, F., 2012. Structural stochastic volatility in asset pricing dynamics: estimation and model contest. J. Econ. Dyn. Control 36 (8), 1193–1211.
Franke, R., Westerhoff, F., 2017. Taking stock: a rigorous modelling of animal spirits in macroeconomics. J. Econ. Surv. 31 (5), 1152–1182.
Galí, J., 2015. Monetary policy, inflation, and the business cycle: an introduction to the New Keynesian framework and its applications. Number 10495 in Economics Books, second ed. Princeton University Press.
Gaunersdorfer, A., Hommes, C., 2007. A Nonlinear Structural Model for Volatility Clustering. Springer Berlin Heidelberg, Berlin, Heidelberg, pp. 265–288.
Gaunersdorfer, A., Hommes, C.H., Wagener, F.O., 2008. Bifurcation routes to volatility clustering under evolutionary learning. J. Econ. Behav. Organ. 67 (1), 27–47.
Gigerenzer, G., Brighton, H., 2009. Homo heuristicus: why biased minds make better inferences. Top. Cogn. Sci. 1 (1), 107–143.
Gourieroux, C., Monfort, A., Renault, E., 1993. Indirect inference. J. Appl. Econom. 8 (S1), S85–S118.
Grazzini, J., Richiardi, M.G., Tsionas, M., 2017. Bayesian estimation of agent-based models. J. Econ. Dyn. Control 77, 26–47.
Guerini, M., Moneta, A., 2017. A method for agent-based models validation. J. Econ. Dyn. Control 82, 125–141.
Haldane, A.G., Madouros, V., 2012. The dog and the frisbee. In: Proceedings, Economic Policy Symposium. Federal Reserve Bank of Kansas City, Jackson Hole.
Hamilton, J.D., 1994. State-space models. Handb. Econom. 4, 3039–3080.
Hastie, T., Tibshirani, R., Friedman, J., 2009. The elements of statistical learning. Springer Series in Statistics, second ed.. New York
Hommes, C., 2013. Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems. Cambridge University Press. Cambridge Books Online
Hommes, C., 2021. Behavioral and experimental macroeconomics and policy analysis: a complex systems approach. J. Econ. Lit. 59 (1), 149–219.
Hommes, C., Makarewicz, T., Massaro, D., Smits, T., 2017. Genetic algorithm learning in a new keynesian macroeconomic setup. J. Evol. Econ. 27 (5), 1133–1155.
Hommes, C., Massaro, D., Weber, M., 2019. Monetary policy under behavioral expectations: theory and experiment. Eur. Econ. Rev. 118, 193–212.
Hommes, C., Sonnemans, J., Tuinstra, J., van de Velden, H., 2005. Coordination of expectations in asset pricing experiments. Rev. Financ. Stud. 18 (3), 955–980.
Hommes, C., Sonnemans, J., Tuinstra, J., van de Velden, H., 2008. Expectations and bubbles in asset pricing experiments. J. Econ. Behav. Organ. 67 (1), 116–133.
Hommes, C.H., 2006. Heterogeneous agent models in economics and finance. In: Tesfatsion, L., Judd, K. (Eds.), Handbook of Computational Economics, Volume 2 of Handbook of Computational Economics. Elsevier, pp. 1109–1186. Chapter 23
Jang, T.S., 2012. Structural estimation of the New-Keynesian model: a formal test of backward-and forward-looking behavior. DSGE Models in Macroeconomics: Estimation, Evaluation, and New Developments. Emerald Group Publishing Limited.
Jang, T.-S., Sacht, S., 2016. Animal spirits and the business cycle: empirical evidence from moment matching. Metroeconomica 67 (1), 76–113.
Jang, T.-S., Sacht, S., 2021. Forecast heuristics, consumer expectations, and New-Keynesian macroeconomics: A Horse race. J. Econ. Behav. Organ. 182, 493–511.
Kleibergen, F., Mavroeidis, S., 2014. Identification issues in limited-information Bayesian analysis of structural macroeconomic models. J. Appl. Econom. 29 (7), 1183–1209.
Kohlscheen, E., Moessner, R., 2022. Globalisation and the slope of the Phillips curve. Econ. Lett. 216, 110587.
Kristensen, D., Shin, Y., 2012. Estimation of dynamic models with nonparametric simulated maximum likelihood. J. Econom. 167 (1), 76–94.
Kukacka, J., Barunik, J., 2017. Estimation of financial agent-based models with simulated maximum likelihood. J. Econ. Dyn. Control 85, 21–45.
Kulish, M., Pagan, A., 2017. Estimation and solution of models with expectations and structural changes. J. Appl. Econom. 32 (2), 255–274.
Lamperti, F., 2018a. Empirical validation of simulated models through the GSL-div: an illustrative application. J. Econ. Interact. Coord. 13 (1), 143–171.
Lamperti, F., 2018b. An information theoretic criterion for empirical validation of simulation models. Econom. Stat. 5, 83–106.
Lamperti, F., Roventini, A., Sani, A., 2018. Agent-based model calibration using machine learning surrogates. J. Econ. Dyn. Control 90, 366–389.
Lee, D., Song, K., 2015. Simulated maximum likelihood estimation for discrete choices using transformed simulated frequencies. J. Econom. 187 (1), 131–153.
Lindé, J., 2005. Estimating New-Keynesian Phillips curves: a full information maximum likelihood approach. J. Monet. Econ. 52 (6), 1135–1149.
Liu, C., Minford, P., 2014. Comparing behavioural and rational expectations for the US post-war economy. Econ. Model. 43, 407–415.
Lubik, T.A., Schorfheide, F., 2004. Testing for indeterminacy: an application to U.S. monetary policy. Am. Econ. Rev. 94 (1), 190–217.
Lux, T., Zwinkels, R.C., 2018. Empirical validation of agent-based models. In: Hommes, C., LeBaron, B. (Eds.), Handbook of Computational Economics, Volume 4 of Handbook of Computational Economics. Elsevier, pp. 437–488. Chapter 8
Martinoli, M., Moneta, A., Pallante, G., 2022. Calibration and Validation of Macroeconomic Simulation Models: A General Protocol by Causal Search. Technical report. Institute of Economics and EMbeDS, Scuola Superiore Sant'Anna.
Milani, F., 2011. Expectation shocks and learning as drivers of the business cycle. Econ. J. 121 (552), 379–401.
Moons, C., Garretsen, H., van Aarle, B., Fornero, J., 2007. Monetary policy in the New-Keynesian model: an application to the Euro area. J. Policy Model. 29 (6), 879–902.
Munier, B., Selten, R., Bouyssou, D., Bourgine, P., Day, R., Harvey, N., Hilton, D., Machina, M.J., Parker, P., Sterman, J., Weber, E., Wernerfelt, B., Wensley, R., 1999. Bounded rationality modeling. Mark. Lett. 10 (3), 233–248.
Nason, J.M., Smith, G.W., 2008. The New Keynesian Phillips curve: lessons from single-equation econometric estimation. Econ. Q. 94 (4), 361–395.
Özden, T., 2021. Heterogeneous Expectations and the Business Cycle at the Effective Lower Bound. Technical report. De Nederlandsche Bank Working Paper.
Schorfheide, F., 2008. DSGE model-based estimation of the New Keynesian Phillips curve. FRB Richmond Econ. Q. 94 (4), 397–433.
Silverman, B.W., 1986. Density Estimation for Statistics and Data Analysis. Chapman and Hall, London.
Simon, H.A., 1972. The Sciences of the Artificial. MIT Press.
Sims, C.A., 1980. Macroeconomics and reality. Econometrica 48 (1), 1–48.
Stock, J.H., Watson, M.W., 1999. Forecasting inflation. J. Monet. Econ. 44 (2), 293–335.
Vandin, A., Giachini, D., Lamperti, F., Chiaromonte, F., 2022. Automated and distributed statistical analysis of economic agent-based models. J. Econ. Dyn. Control 143, 104458 .