Observables are proper models of measurements

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Abstract

A quantitative observation assigns numerical values to a phenomenon, $p \in \boldsymbol{p}$ (e.g. a system's property). To ensure a proper observation process, any hidden feedback must be avoided. It means that the uncertainty, $u \in \boldsymbol{u} \neq \emptyset$**,** affecting the assignment must not depend on the phenomenon itself. Since quantification implicitly involves comparisons (e.g. "a is smaller than b", "c is more desired than d", etc.), it assumes the existence of a transitive and complete ordering $\preccurlyeq$ on $\boldsymbol{p}$**.** It can be shown, that its completeness is always attainable under uncertainty.

The result [1] implies existence of a continuous, ordering-preserving, quantitative observation iff the topology of open intervals in $(\prec,\ \boldsymbol{p})$ does not require more complexity than the natural ordering of real numbers. Hence, it is possible to distinguish a *countable* number of realizations of the quantitatively-described phenomenon and a *countable* number of uncertainties that can be associated.

Therefore, the observation mapping, $\mathcal{O}: (\boldsymbol{p}, \boldsymbol{u}) \mapsto \boldsymbol{o}$, has a matrix structure, $\mathcal{O} = \left[O_{(p,u)}\right]$, $p \in \boldsymbol{p}, u \in \boldsymbol{u}$. To mitigate the influence of indices corresponding to phenomenon and uncertainty, the singular value decomposition (SVD) is applied, $O = SVN^*$ with N^* denoting transposition and conjugation of N, [2]. Structurally, this implies that the uncertainty-modelling unitary matrix N spans complex Hilbert's space. Subspaces of this space are projected onto quantitative observations in $\boldsymbol{o}$. These subspaces represent the relevant, distinguishable random events. Thus, the quantitative observation is to be handled as an observable [3]. The proposed work elaborates on and discusses this idea.

The twin work [4] addresses this viewpoint within the context of decision making. It demonstrates that a probabilistic model applied to subspaces modelling uncertainties is appropriate. The present study suggests that the findings of [4] are applicable to any quantitative observation (measurement).

Keywords measurement, quantum model of uncertain events, decision making

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Acknowledgement EU-COST Action CA21169 supports this research.