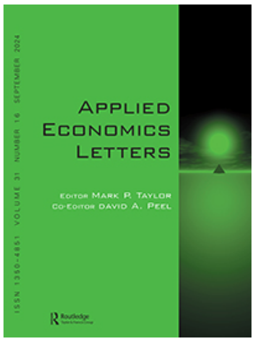

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Beyond GARCH in cryptocurrency volatility modelling: superiority of range-based estimators

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ABSTRACT
Cryptoassets are extremely volatile with possible volatility jumps and infrastructure noise, making the estimation of true volatility process challenging. When the high-frequency data are not available, the true volatility needs to be estimated to be further studied or forecasted. The GARCH-family models have become a norm in the field. Here, we examine the performance of 6 GARCH-type specifications with 4 distributional assumptions and compare them with 4 non-parametric range-based models built on the daily 'candles'. Our study focuses on five popular cryptocurrencies (Bitcoin, Ethereum, BNB, XRP, and Dogecoin) between 1 July 2019 and 30 September 2022, utilizing Binance 5-minute data for realized measures as the high-frequency estimators of the true volatility process. The results reveal that the Garman-Klass estimator clearly outperforms the GARCH-family models in all studied settings, and the other range-based estimators remain competitive with the GARCH-family models. These results are crucial for studies on volatility in cryptoassets where using the GARCH-type models is a standard. When the high-frequency data are not available, the range-based estimators, and the Garman-Klass estimator in particular, should be preferred as proxies for the true volatility process over the GARCH-type models, be it in the in-sample, more qualitative studies, or the forecasting, out-of-sample exercises.

KEYWORDS
Cryptoasset; cryptocurrency; volatility; GARCH; Garman-Klass

JEL CLASSIFICATION
C32; C53; G23

I. Introduction

Volatility of cryptoassets,[1] and initially mostly Bitcoin, has been the topic of interest since the very beginnings of the crypto-related financial and economic research, mostly due to its unprecedented levels which have obvious implications for portfolio diversification and risk management (Sapuric and Kokkinaki 2014). Even though data availability is one of the often cited features and advantages of cryptocurrencies, utilizing high-frequency data for estimating integrated (true) volatility is not the dominant approach (Kim, Trimborn, and Härdle 2021). The GARCH-family models are established as the prevalent method for estimating daily volatility (Bouri, Lucey, and Roubaud 2020; Ghosh et al. 2023). However, it is not clear whether these types of models are suitable for estimating the true volatility process in assets with such high levels of volatility, likely intra-daily jumps, microstructure noise, and frequent runs and rebounds during their sessions, more so when the crypto-markets are open 24/7. Precise and efficient estimation of the true volatility process is crucial for assessing performance of predictive models as well as evaluation of dynamic properties for the emergent class of cryptoassets (Almeida and Goncalves 2022; Hattori 2020). What use would a GARCH-type model be if it predicts the future GARCH-volatility precisely but that future GARCH-volatility does not match or is not close to the true volatility process?

Here, we provide a detailed evaluation of the GARCH-family models and their ability to fit the integrated (true) volatility in the top cryptoassets and contrasted with representatives of the range-based estimators. The results suggest and we argue that the range-based estimators in cryptoassets should be more deeply studied and more frequently utilized as estimates of volatility when high-frequency data are unavailable.

II. Methods and data

The family of the generalized autoregressive conditional heteroskedasticity models (GARCH) is wide and rich, controlling for various types of stylized

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[1]We use the terms cryptoassets and cryptocurrencies interchangeably throughout the text.

facts in financial volatility (Cont 2001) such as clustering, leverage effects and asymmetry, and extreme events (Gao, Zhang, and Zhang 2012; Katsiampa 2017). We build on the baseline standard GARCH (1,1) that reflects on the strong autocorrelation structure and clustering in volatility. We utilize four distributions of the error term, namely the normal, Student-t, skewed Student-t, and generalized error distribution. However, the financial volatility processes possess various peculiar properties which lead us to including additional GARCH-family specifications:

- GJR-GARCH (Glosten, Jagannathan, and Runkle 1993): possible asymmetry of positive and negative shocks into the volatility process;
- Integrated GARCH (IGARCH) (Engle and Bollerslev 1986): non-stationary, persistent variance process;
- Exponential GARCH (EGARCH) (Nelson 1991): asymmetric effects in logarithmic volatility;
- Component GARCH (CGARCH) (Lee and Engle 1999): decomposing the conditional variance into the long- (permanent) and short-term (transitory) factors;
- Asymmetric power ARCH (APARCH) (Ding 1993): leverage effect and persistence via the Box-Cox transformation of the volatility process

For each model type, we work with the simplest parameter specification such as GJR-GARCH(1,1), following Hansen and Lunde (2005). We thus work with 6 GARCH-family models, each with 4 distributional assumptions, i.e. 24 models in total.

The GARCH-family models do not use the standardly available complete 'candles' of the daily price series (open, close, high, and low prices for the given day). The range-based estimators form a non-parametric family of daily variance estimators based solely on the 'candles'. As the cryptocurrencies are traded in a 24/7 fashion, several issues connected to the market opening and closing effects, and non-trading sessions are not in place. We work with the Parkinson (1980) and Garman and Klass (1980) estimators as representatives of the range-based estimators. As the simplest variants of the range-based estimators, we also include the absolute and squared returns, i.e. $|r|$ and r^2, respectively, using the open-close specification.

As the true (integrated) volatility is not directly observable, we utilize its direct estimators based on the high-frequency data (McAleer and Medeiros 2008): realized variance – RV, bipower variation (less sensitive to microstructure noise, specifically jumps) – BV, and realized kernel (less sensitive to other specific types of microstructure noise, mostly the bid-ask bounce) – RK.

A set of 5 popular cryptocurrencies is analyzed, each representing different sectors of the market – Bitcoin (BTC), Ethereum (ETH), BNB (BNB), XRP (XRP), and Dogecoin (DOGE) – between 1 July 2019 and 30 September 2022, utilizing the data from Binance API.[2] The daily returns utilized in the GARCH models are formed as logarithmic open-close returns. Overall, we work with 1188 daily returns and 342,144 5-min returns for each examined cryptoasset.

III. Results and discussion

To investigate the performance of the volatility models and estimators more properly, we utilize the Mincer-Zarnowitz regression (Mincer and Zarnowitz 1969) which reads simply

$$\widehat{IV}_t = \alpha + \beta\widehat{\sigma}_t + \epsilon_t \qquad (1)$$

where $\widehat{IV}$ is the estimate of the true (integrated) volatility (the realized measures) and $\widehat{\sigma}$ are the estimated volatilities based on either the GARCH-family models or the range-based estimators, expecting $\alpha = 0$ and $\beta = 1$ for an unbiased and accurate estimator, respectively. R^2 of the regression is an essential part of information about the estimator quality as it says how much of the overall dynamics of the true volatility process can be explained by the given estimator.

Table 1 summarizes the results for the realized volatility as an approximation of the true volatility process.[3] We report results only for the assumptions for each of the 6 GARCH-type models to make the

[2] Currently freely available at https://data.binance.vision/?prefix=data/spot/daily/klines/.

[3] The results for the bipower variation and the realized kernel are available in Tables A1 and Table A2. in the Appendix. The implications are qualitatively parallel to the realized volatility case.

Table 1. Mincer-Zarnowiz regression results - realized volatility.

Model	$\hat{\alpha}$	$\hat{\beta}$	SE($\hat{\alpha}$)	SE($\hat{\beta}$)	R^2	*F*-test	*p*-value
Bitcoin							
GARCH − norm	−0.0019	2.2886	0.0002	0.1134	0.2557	65.9370	$\ll 0.0001$
GJR-GARCH − norm	−0.0007	1.4194	0.0001	0.0428	0.3446	1.5455	0.2186
IGARCH − norm	−0.0001	0.9084	0.0001	0.0448	0.2571	4.5106	0.0112
EGARCH − norm	−0.0020	2.3793	0.0002	0.0921	***0.3601***	114.1000	$\ll 0.0001$
CGARCH − norm	−0.0021	2.3704	0.0002	0.1212	0.2437	65.1720	$\ll 0.0001$
APARCH − norm	0.0007	0.5930	0.0001	0.0243	0.3334	140.3100	$\ll 0.0001$
Garman-Klass	0.0001	0.9234	0.0000	0.0092	**0.8938**	34.4980	< 0.0001
Parkinson	0.0005	0.7582	0.0001	0.0146	0.6950	137.5500	$\ll 0.0001$
r^2	0.0014	0.2635	0.0001	0.0150	0.2065	1207.7000	$\ll 0.0001$
$\|r\|$	−0.0001	0.0731	0.0002	0.0039	0.2268	50789.0000	$\ll 0.0001$
Ethereum							
GARCH − norm	0.0000	0.9768	0.0003	0.0625	0.1709	0.0961	0.9084
GJR-GARCH − ged	0.0000	1.0570	0.0002	0.0612	0.2013	0.8610	0.4230
IGARCH − std	−0.0003	0.5688	0.0002	0.0290	0.2448	247.3000	$\ll 0.0001$
EGARCH − norm	−0.0008	0.9496	0.0001	0.0359	***0.3710***	43.8390	$\ll 0.0001$
CGARCH − std	−0.0013	1.2572	0.0002	0.0600	0.2705	25.6820	< 0.0001
APARCH − norm	0.0000	0.6321	0.0001	0.0262	0.3296	151.1500	$\ll 0.0001$
Garman-Klass	0.0000	0.9391	0.0001	0.0092	**0.8987**	24.3750	< 0.0001
Parkinson	0.0005	0.7993	0.0001	0.0137	0.7413	107.2600	$\ll 0.0001$
r^2	0.0020	0.3011	0.0002	0.0149	0.2563	1101.2000	$\ll 0.0001$
$\|r\|$	−0.0005	0.0935	0.0002	0.0046	0.2612	38079.0000	$\ll 0.0001$
BNB							
GARCH − std	0.0001	0.9685	0.0002	0.0373	0.3628	0.3611	0.6970
GJR-GARCH − norm	0.0005	0.7462	0.0002	0.0284	***0.3680***	42.4810	$\ll 0.0001$
IGARCH − std	0.0007	0.2668	0.0001	0.0163	0.1837	1188.2000	$\ll 0.0001$
EGARCH − norm	0.0004	0.4162	0.0001	0.0193	0.2823	559.5100	$\ll 0.0001$
CGARCH − std	0.0005	0.4239	0.0001	0.0242	0.2055	338.3800	$\ll 0.0001$
APARCH − norm	0.0008	0.2847	0.0001	0.0141	0.2566	1432.6000	$\ll 0.0001$
Garman-Klass	0.0002	0.9028	0.0001	0.0078	**0.9193**	79.6170	$\ll 0.0001$
Parkinson	0.0006	0.4268	0.0001	0.5702	0.8252	3154.3000	$\ll 0.0001$
r^2	0.0021	0.3152	0.0002	0.0127	0.3414	1451.4000	$\ll 0.0001$
$\|r\|$	−0.0009	0.1132	0.0002	0.0044	0.3629	35546.0000	$\ll 0.0001$
XRP							
GARCH − norm	0.0013	0.6910	0.0003	0.0321	***0.2811***	46.4290	$\ll 0.0001$
GJR-GARCH − sstd	0.0001	0.9720	0.0004	0.0480	0.2567	0.1705	0.8433
IGARCH − norm	0.0012	0.1393	0.0001	0.0144	0.0729	2017.2000	$\ll 0.0001$
EGARCH − norm	0.0009	0.2308	0.0001	0.0185	0.1157	1030.6000	$\ll 0.0001$
CGARCH − ged	0.0009	0.2429	0.0002	0.0223	0.0906	688.4100	$\ll 0.0001$
APARCH − sstd	0.0010	0.1743	0.0002	0.0215	0.0527	984.3700	$\ll 0.0001$
Garman-Klass	0.0007	0.7948	0.0001	0.0091	**0.8644**	253.7800	$\ll 0.0001$
Parkinson	0.0010	0.3851	0.0002	0.5394	0.8113	2880.2000	$\ll 0.0001$
r^2	−0.0013	0.1534	0.0003	0.0053	0.4124	21088.0000	$\ll 0.0001$
$\|r\|$	0.0027	0.4703	0.0003	0.0167	0.4011	508.1900	$\ll 0.0001$
Dogecoin							
GARCH − ged	0.0024	0.6297	0.0006	0.0147	***0.6092***	319.0700	$\ll 0.0001$
GJR-GARCH − ged	0.0023	0.6647	0.0006	0.0155	0.6079	233.2900	$\ll 0.0001$
IGARCH − norm	0.0015	0.0346	0.0001	0.0074	0.0181	9470.0000	$\ll 0.0001$
EGARCH − ged	0.0018	0.0001	0.0001	0.0000	0.0072	568716299.0000	$\ll 0.0001$
CGARCH − ged	0.0017	0.0152	0.0001	0.0032	0.0184	47074.0000	$\ll 0.0001$
APARCH − ged	0.0017	0.0160	0.0001	0.0036	0.0163	38274.0000	$\ll 0.0001$
Garman-Klass	−0.0008	1.1507	0.0004	0.0126	**0.8765**	72.3490	$\ll 0.0001$
Parkinson	0.0024	0.7047	0.0006	0.0174	0.5808	143.9600	$\ll 0.0001$
r^2	0.0061	0.1855	0.0009	0.0114	0.1830	2550.7000	$\ll 0.0001$
$\|r\|$	−0.0033	0.2630	0.0009	0.0108	0.3346	3209.9000	$\ll 0.0001$

Table 2. Basic descriptive statistics of volatility processes.

Cryptoasset	Model	Mean	SD	Min	Max
BTC	EGARCH − norm	0.0016	0.0012	0.0008	0.0255
	Garman-Klass	0.0019	0.0047	0.0000	0.0943
	realized volatility	0.0018	0.0046	0.0001	0.1099
	bipower variation	0.0011	0.0029	0.0000	0.0631
	realized kernel	0.0017	0.0044	0.0000	0.1157
ETH	EGARCH − norm	0.0028	0.0029	0.0011	0.0638
	Garman-Klass	0.0030	0.0069	0.0001	0.1395
	realized volatility	0.0028	0.0069	0.0001	0.1634
	bipower variation	0.0017	0.0046	0.0001	0.1168
	realized kernel	0.0027	0.0059	0.0001	0.1295

(*Continued*)

Table 2. (Continued).

Cryptoasset	Model	Mean	SD	Min	Max
BNB	GJR-GARCH – norm	0.0036	0.0066	0.0008	0.0921
	Garman-Klass	0.0032	0.0087	0.0001	0.1659
	realized volatility	0.0031	0.0082	0.0002	0.1672
	bipower variation	0.0019	0.0053	0.0001	0.1049
	realized kernel	0.0031	0.0082	0.0002	0.2009
XRP	GARCH – norm	0.0046	0.0088	0.0009	0.1173
	Garman-Klass	0.0047	0.0134	0.0000	0.2436
	realized volatility	0.0045	0.0115	0.0001	0.2042
	bipower variation	0.0027	0.0074	0.0000	0.1505
	realized kernel	0.0043	0.0107	0.0001	0.1713
DOGE	GARCH – ged	0.0079	0.0416	0.0008	1.0290
	Garman-Klass	0.0071	0.0273	0.0000	0.5078
	realized volatility	0.0074	0.0336	0.0002	0.8717
	bipower variation	0.0045	0.0221	0.0000	0.5702
	realized kernel	0.0070	0.0308	0.0002	0.7700

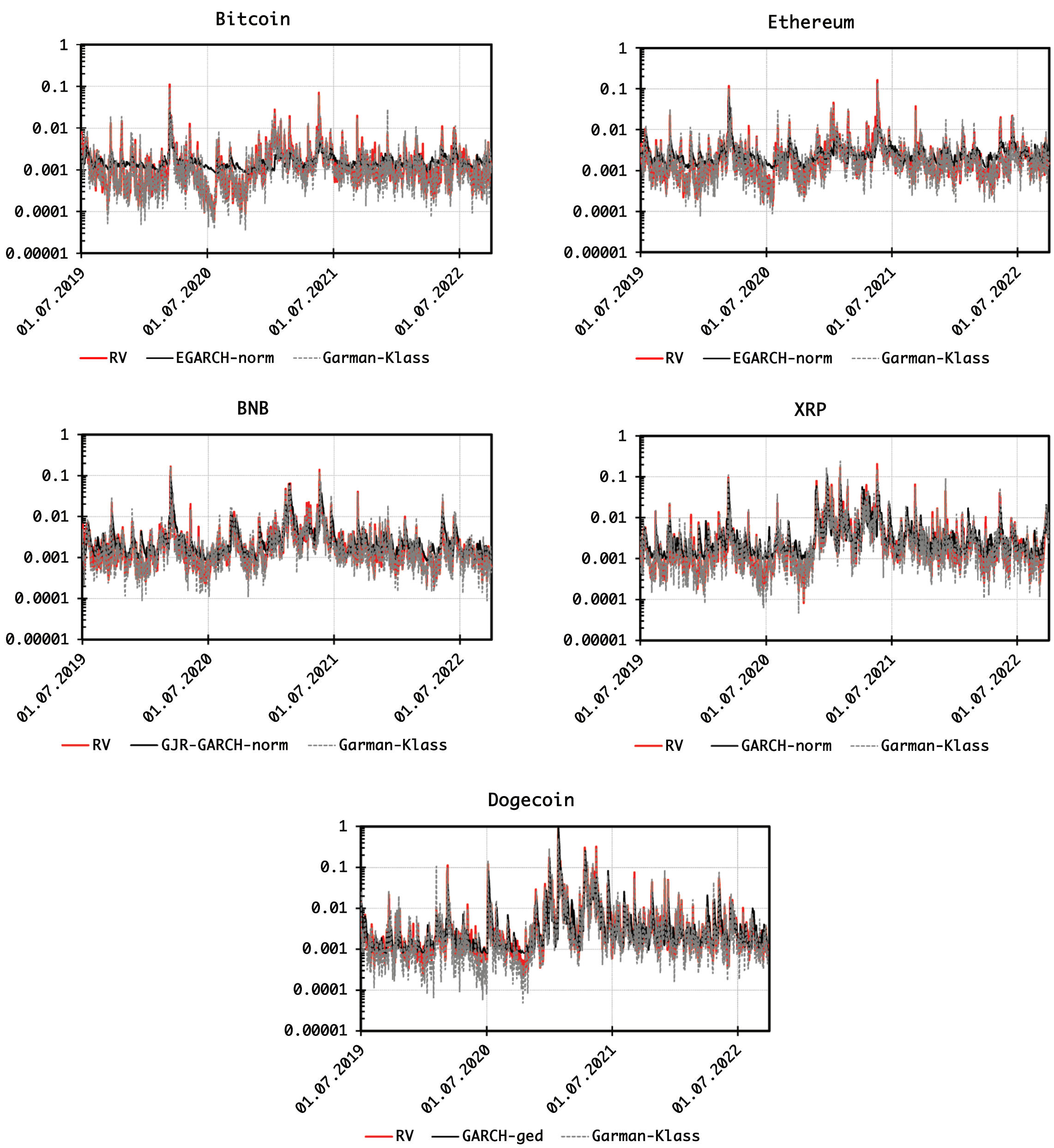

Figure 1. Comparison of the best models and estimators with the true volatility process.

table easy to read. In the table and for each cryptoasset, the best-performing model is presented in bold and the best-performing GARCH-family model is shown in italics. A broader context is provided by Table 2, listing the basic descriptive statistics for the realized measures and the best-performing of the GARCH-family models and the range-based estimators, and Figure 1, plotting the dynamics of these (only the realized volatility from the realized measures is shown for better legibility). Putting these together gives the following picture.

First and foremost, the range-based estimators mostly dominate the GARCH-family models. The Garman-Klass estimator emerges as a clear winner by a marked margin in R^2. Apart from DOGE, even the Parkinson estimator clearly beats the GARCH models. And for most cases, even the absolute and squared returns are competitive compared to the GARCH models. Second, the GARCH models overestimate the real volatility in the tranquil periods. This is clear from Figure 1 and mostly visible for Bitcoin and Ethereum, and also reflected in much higher minimum values of estimated volatility compared to the realized measures, as shown in Table 2. The GARCH models also underestimate the most risky periods, reflected in lower maximum values. The rule seems to be: the higher the volatility of the cryptoasset, the less underrepresented the tranquil and the extreme periods are in the GARCH models. The GARCH-models implied volatility is also smoother than the real volatility. Third, the Garman-Klass estimator is very close to the realized measures in the basic descriptives with the only exception of DOGE where the dominance of the range-based estimators is not as straightforward. All in all, the range-based estimators are found highly competitive for estimating the true volatility process in cryptocurrencies compared to the traditional GARCH-family models. Specifically the Garman-Klass estimator outperforms the GARCH models by a league. Even the Parkinson estimator is highly competitive. The results suggest that the range-based estimators should be preferred over the GARCH-family models when finding a proxy for the true volatility process in the absence or unavailability of the high-frequency data needed for the realized measures. The GARCH-type models tend to smooth out the volatility process excessively, suppressing its dynamic properties, and underrepresenting both the most tranquil and the most erratic periods in cryptocurrencies.

IV. Conclusions

The current study provides a deep insight into the GARCH models accuracy and brings up a straightforward message that the GARCH-family models are not competitive in estimating the true volatility process compared to the range-based estimators, specifically the Garman-Klass estimator utilizing the always available daily 'candles' (open-close-high-low prices), which clearly dominates the GARCH-family models. Even the simpler Parkinson estimator built on the high-low prices strongly outperforms the GARCH models in most cases. And almost surprisingly, even the absolute and squared returns are comparable to the GARCH models, and in some case even outperform them. These results are crucial for studies on volatility in cryptoassets where using the GARCH-type models is a norm. When the high-frequency data are not available, the range-based estimators, and the Garman-Klass estimator in particular, should be preferred as proxies for the true volatility process over the GARCH-type models, be it in the in-sample, more qualitative studies, or the forecasting, out-of-sample exercises.

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Appendix

Table A1. Mincer-Zarnowiz regression results – bipower variation.

Model	$\hat{\alpha}$	$\hat{\beta}$	SE($\hat{\alpha}$)	SE($\hat{\beta}$)	R^2	*F*-test	*p*-value
Bitcoin							
GARCH – norm	−0.0011	1.3314	0.0001	0.0732	0.2179	38.8420	$\ll$ 0.0001
GJR-GARCH – norm	0.0000	0.6197	0.0001	0.0281	0.2914	137.5400	$\ll$ 0.0001
IGARCH – norm	0.0000	0.5286	0.0001	0.0290	0.2191	223.7000	$\ll$ 0.0001
EGARCH – norm	−0.0012	1.3864	0.0001	0.0604	***0.3078***	50.1210	$\ll$ 0.0001
CGARCH – norm	−0.0012	1.3773	0.0001	0.0782	0.2072	39.9890	$\ll$ 0.0001
APARCH – norm	0.0004	0.3424	0.0001	0.0160	0.2799	924.0600	$\ll$ 0.0001
Garman-Klass	0.0000	0.5752	0.0000	0.0064	**0.8730**	2559.1000	$\ll$ 0.0001
Parkinson	0.0002	0.4700	0.0001	0.0095	0.6722	1654.4000	$\ll$ 0.0001
r^2	0.0008	0.1608	0.0001	0.0095	0.1935	3901.8000	$\ll$ 0.0001
$\lvert r\rvert$	−0.0001	0.0445	0.0001	0.0025	0.2117	133166.0000	$\ll$ 0.0001
Ethereum							
GARCH – std	−0.0006	0.8399	0.0002	0.0555	0.1616	36.0460	< 0.0001
GJR-GARCH – ged	−0.0001	0.6774	0.0002	0.0417	0.1819	60.0080	$\ll$ 0.0001
IGARCH – std	−0.0002	0.3339	0.0001	0.0187	0.2124	1275.9000	$\ll$ 0.0001
EGARCH – norm	−0.0005	0.5546	0.0001	0.0236	***0.3186***	489.5900	$\ll$ 0.0001
CGARCH – std	−0.0007	0.7378	0.0001	0.0387	0.2345	205.3100	$\ll$ 0.0001
APARCH – norm	0.0000	0.3680	0.0001	0.0171	0.2814	1028.1000	$\ll$ 0.0001
Garman-Klass	−0.0001	0.6211	0.0001	0.0071	**0.8650**	1730.7000	$\ll$ 0.0001
Parkinson	0.0002	0.5193	0.0001	0.0101	0.6887	1243.9000	$\ll$ 0.0001
r^2	0.0012	0.1855	0.0001	0.0103	0.2140	3146.8000	$\ll$ 0.0001
$\lvert r\rvert$	−0.0004	0.0589	0.0002	0.0031	0.2281	86090.0000	$\ll$ 0.0001
BNB							
GARCH – std	0.0000	0.6134	0.0001	0.0243	0.3498	175.3700	$\ll$ 0.0001
GJR-GARCH – norm	0.0003	0.4720	0.0001	0.0185	***0.3540***	493.9100	$\ll$ 0.0001
IGARCH – std	0.0004	0.1591	0.0001	0.0104	0.1645	4042.6000	$\ll$ 0.0001
EGARCH – norm	0.0002	0.2458	0.0001	0.0124	0.2478	2360.9000	$\ll$ 0.0001
CGARCH – std	0.0003	0.2519	0.0001	0.0155	0.1827	1514.8000	$\ll$ 0.0001
APARCH – norm	0.0005	0.1682	0.0001	0.0091	0.2255	4877.1000	$\ll$ 0.0001
Garman-Klass	0.0001	0.5783	0.0000	0.0054	**0.9067**	3458.6000	$\ll$ 0.0001
Parkinson	0.0003	27.1900	0.0001	0.3882	0.8053	2664.6000	$\ll$ 0.0001
r^2	0.0013	0.1982	0.0001	0.0083	0.3245	4709.3000	$\ll$ 0.0001
$\lvert r\rvert$	−0.0006	0.0717	0.0002	0.0028	0.3493	90951.0000	$\ll$ 0.0001
XRP							
GARCH – norm	0.0008	0.4306	0.0002	0.0210	***0.2621***	418.2000	$\ll$ 0.0001
GJR-GARCH – sstd	0.0000	0.6048	0.0002	0.0314	0.2386	123.4100	$\ll$ 0.0001
IGARCH – norm	0.0007	0.0831	0.0001	0.0091	0.0653	5996.8000	$\ll$ 0.0001
EGARCH – norm	0.0005	0.1369	0.0001	0.0118	0.1024	3427.3000	$\ll$ 0.0001
CGARCH – ged	0.0005	0.1464	0.0001	0.0141	0.0830	2373.0000	$\ll$ 0.0001
APARCH – sstd	0.0006	0.1066	0.0001	0.0136	0.0496	3149.9000	$\ll$ 0.0001
Garman-Klass	0.0004	0.4965	0.0001	0.0070	**0.8098**	2820.8000	$\ll$ 0.0001
Parkinson	0.0005	24.0500	0.0001	0.3930	0.7594	2034.9000	$\ll$ 0.0001
r^2	0.0017	0.2933	0.0002	0.0110	0.3745	2078.3000	$\ll$ 0.0001
$\lvert r\rvert$	−0.0009	0.0956	0.0002	0.0035	0.3843	54511.0000	$\ll$ 0.0001
Dogecoin							
GARCH – ged	0.0012	0.4163	0.0004	0.0096	***0.6161***	1900.7000	$\ll$ 0.0001
GJR-GARCH – ged	0.0011	0.4394	0.0004	0.0101	0.6147	1565.9000	$\ll$ 0.0001
IGARCH – norm	0.0009	0.0227	0.0001	0.0047	0.0196	25057.0000	$\ll$ 0.0001
EGARCH – ged	0.0011	0.0001	0.0001	0.0000	0.0081	1432000000.0000	$\ll$ 0.0001
CGARCH – ged	0.0010	0.0098	0.0001	0.0020	0.0191	120413.0000	$\ll$ 0.0001
APARCH – ged	0.0010	0.0101	0.0001	0.0023	0.0164	98116.0000	$\ll$ 0.0001
Garman-Klass	−0.0008	0.7532	0.0002	0.0085	**0.8689**	482.0600	$\ll$ 0.0001
Parkinson	0.0012	0.4628	0.0004	0.0115	0.5795	1117.2000	$\ll$ 0.0001
r^2	0.0036	0.1226	0.0006	0.0075	0.1850	6874.4000	$\ll$ 0.0001
$\lvert r\rvert$	−0.0025	0.1716	0.0006	0.0071	0.3296	9144.9000	$\ll$ 0.0001

Table A2. Mincer-Zarnowiz regression results – realized kernel.

Model	$\widehat{\alpha}$	$\widehat{\beta}$	SE($\widehat{\alpha}$)	SE($\widehat{\beta}$)	R^2	F-test	p-value
Bitcoin							
GARCH – norm	−0.0021	2.3284	0.0002	0.1057	0.2903	79.2440	≪ 0.0001
GJR-GARCH – norm	−0.0002	1.1014	0.0001	0.0391	0.4011	3.4318	0.0327
IGARCH – norm	−0.0002	0.9245	0.0001	0.0418	0.2921	7.3526	0.0007
EGARCH – norm	−0.0022	2.4362	0.0002	0.0842	***0.4140***	146.1700	≪ 0.0001
CGARCH – norm	−0.0022	2.4004	0.0002	0.1134	0.2741	76.4720	≪ 0.0001
APARCH – norm	0.0005	0.6127	0.0001	0.0222	0.3905	154.3400	≪ 0.0001
Garman-Klass	0.0001	0.8708	0.0000	0.0097	**0.8718**	93.2200	≪ 0.0001
Parkinson	0.0005	0.7109	0.0001	0.0145	0.6698	199.2600	≪ 0.0001
r^2	0.0013	0.2416	0.0001	0.0145	0.1903	1374.4000	≪ 0.0001
$\|r\|$	−0.0001	0.0681	0.0002	0.0038	0.2161	55457.0000	≪ 0.0001
Ethereum							
GARCH – std	−0.0010	1.3592	0.0002	0.0665	0.2605	14.6060	< 0.0001
GJR-GARCH – ged	−0.0003	1.1129	0.0002	0.0491	0.3022	2.6489	0.0711
IGARCH – std	−0.0004	0.5746	0.0001	0.0272	0.2741	297.0900	≪ 0.0001
EGARCH – norm	−0.0010	0.9727	0.0001	0.0327	***0.4270***	64.1010	≪ 0.0001
CGARCH – std	−0.0014	1.2708	0.0002	0.0560	0.3031	37.4870	≪ 0.0001
APARCH – norm	−0.0002	0.6466	0.0001	0.0241	0.3784	182.4300	≪ 0.0001
Garman-Klass	0.0003	0.7971	0.0001	0.0087	**0.8770**	285.2100	≪ 0.0001
Parkinson	0.0007	0.6835	0.0001	0.0119	0.7343	353.9400	≪ 0.0001
r^2	0.0020	0.2630	0.0002	0.0127	0.2649	1677.5000	≪ 0.0001
$\|r\|$	−0.0003	0.0831	0.0002	0.0039	0.2796	53763.0000	≪ 0.0001
BNB							
GARCH – std	−0.0001	1.0041	0.0002	0.0371	0.3814	175.3700	≪ 0.0001
GJR-GARCH – norm	0.0003	0.7811	0.0002	0.0281	***0.3944***	493.9100	≪ 0.0001
IGARCH – std	0.0006	0.2702	0.0001	0.0154	0.2068	1350.2000	≪ 0.0001
EGARCH – norm	0.0003	0.4225	0.0001	0.0179	0.3190	651.8500	≪ 0.0001
CGARCH – std	0.0004	0.4318	0.0001	0.0227	0.2339	387.5200	≪ 0.0001
APARCH – norm	0.0006	0.2894	0.0001	0.0131	0.2908	1646.4000	≪ 0.0001
Garman-Klass	0.0001	0.9073	0.0001	0.0084	**0.9080**	3458.6000	≪ 0.0001
Parkinson	0.0005	41.9300	0.0001	0.6480	0.7793	2348.6000	≪ 0.0001
r^2	0.0021	0.2926	0.0002	0.0134	0.2878	1400.7000	≪ 0.0001
$\|r\|$	−0.0008	0.1075	0.0003	0.0046	0.3199	32885.0000	≪ 0.0001
XRP							
GARCH – norm	0.0012	0.6836	0.0003	0.0289	***0.3199***	60.4300	≪ 0.0001
GJR-GARCH – sstd	0.0000	0.9499	0.0003	0.0437	0.2850	0.9883	0.3725
IGARCH – norm	0.0011	0.1414	0.0001	0.0137	0.0824	2246.6000	≪ 0.0001
EGARCH – norm	0.0007	0.2356	0.0001	0.0175	0.1322	1158.2000	≪ 0.0001
CGARCH – ged	0.0008	0.2438	0.0001	0.0212	0.1002	776.2600	≪ 0.0001
APARCH – sstd	0.0009	0.1715	0.0002	0.0205	0.0559	1109.8000	≪ 0.0001
Garman-Klass	0.0007	0.7589	0.0001	0.0067	**0.9162**	666.1800	≪ 0.0001
Parkinson	0.0009	36.7700	0.0001	0.4310	0.8599	4102.7000	≪ 0.0001
r^2	0.0026	0.4491	0.0002	0.0152	0.4252	663.1100	≪ 0.0001
$\|r\|$	−0.0012	0.1464	0.0003	0.0048	0.4367	25956.0000	≪ 0.0001
Dogecoin							
GARCH – ged	0.0026	0.5585	0.0006	0.0141	***0.5697***	490.0900	≪ 0.0001
GJR-GARCH – ged	0.0025	0.5896	0.0006	0.0149	0.5684	377.9500	≪ 0.0001
IGARCH – norm	0.0014	0.0345	0.0001	0.0071	0.0198	10442.0000	≪ 0.0001
EGARCH – ged	0.0017	0.0001	0.0001	0.0000	0.0098	625261182.0000	≪ 0.0001
CGARCH – ged	0.0016	0.0163	0.0001	0.0031	0.0230	51791.0000	≪ 0.0001
APARCH – ged	0.0016	0.0169	0.0001	0.0034	0.0199	42092.0000	≪ 0.0001
Garman-Klass	−0.0006	1.0654	0.0003	0.0107	**0.8930**	18.6250	< 0.0001
Parkinson	0.0021	0.6910	0.0005	0.0143	0.0179	233.4100	≪ 0.0001
r^2	0.0056	0.2023	0.0008	0.0100	0.2585	3204.5000	≪ 0.0001
$\|r\|$	−0.0037	0.2631	0.0008	0.0094	0.3980	4242.7000	≪ 0.0001