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On underactuated bipedal systems walking: Gait pattern modeling and analysis of a stepladder with decorator

P. Polach[a], R. Prokýšek[b], Š. Papáček[c]

[a] Research and Testing Institute Plzen Ltd., Tylova 1581/46, 301 00 Plzeň, Czech Republic
[b] The Department of Mechanics, Biomechanics and Mechatronics, CTU, 160 00 Prague, Czech Republic
[c] The Czech Academy of Sciences, Institute of Information Theory and Automation, Pod vodárenskou věží 4, 182 08 Prague, Czech Republic

1. Introduction

Modeling and analysis of cyclic walking of underactuated bipedal systems play an important role in various applications of control of bipedal robots and rehabilitation robotic devices [1]. In this context, the simplest robot able to walk is the two-link Compass gait walker [6]. Recall here, that the underactuated hybrid mechanical systems where the DOF is bigger than the number of actuated links and the interaction between the continuous and discrete parts of the corresponding mathematical model takes place, are still in the scope of our interest [4][1].

This paper presents how to model one specific, well-known underactuated mechanical system, a stepladder with an operator inducing the motion, and how to analyze the stability of its corresponding gait pattern. Conversely to Compass gait and other common bipedal robotic systems, the leg order is preserved during walking, i.e., the swing leg never overcomes the stance leg. Let us underline three key features of this planar or 2D system, enabling the overall ordered cyclic displacement, e.g., from left to right, i.e., in the positive direction of x-axis in an inertial (Cartesian) system of coordinates:

- The inclination of the decorator helps to diminish the normal (here vertical) force acting on the tip of further swing (either the forward or backward) leg.
- The sudden return of the decorator to the originally vertical position creates an impulsive (inertial) force (momentum) acting on the operator center of mass (against, helping to diminish the normal force acting on the tip of further swing leg).
- Once per cycle, the operator's forward leg forces the stepladder's forward leg to move in the direction of movement (again *via* a strike, modeled as an impulsive force).

The rigorous dynamical analysis of stable cyclic walking of this special class of stepladder models with the periodic movement of an operator (decorator) is presented in the next sections.

2. Model formulation

The stepladder model with a decorator in the double support position is schematically depicted in Fig. 1. The system is represented by the same kinematic scheme as an Acrobot or Compass gait [6] with a third link called an upper body or a torso. This three-link planar mechanism model has two rigid legs, each one with a lumped mass m_2 and m_3, respectively (no inertia),

[1] For a review on the control of underactuated or even passive (non-actuated at all) mechanical systems, see e.g., [3] and [7], respectively. For a study of asymptotically stable walking of biped robots, see [2].

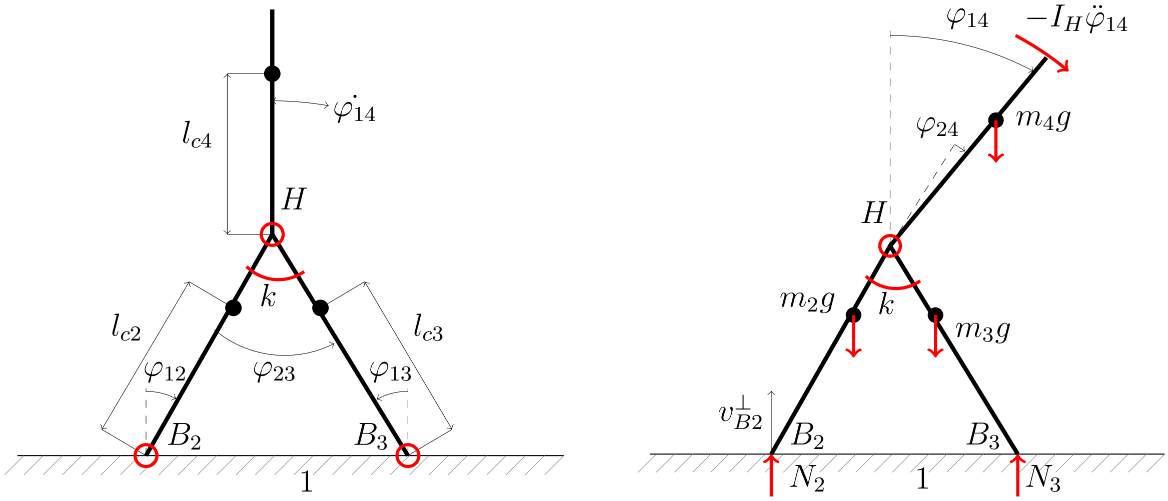

Fig. 1. The stepladder model with a decorator: parameters, coordinates, and forces. (*left*) The initial position for the walking simulation, double support, the link 4 is rotating clockwise; (*right*) initial position of the single support phase. Link 4 is in its right extreme position when $\dot{\varphi}_{14} = 0 \ \wedge \ \ddot{\varphi}_{14} > 0$, i.e., it is just turning back rotating counterclockwise. The reaction N_2 diminishes until 0 and the perpendicular projection of velocity of point B_2 is positive

connected at the revolute (hip) joint H, and the moment of inertia of a torso (the operator) w.r.t. point H is I_H. It has, in general, four degrees of freedom in 2-dimensional space when the "running mode" is not allowed. More precisely, using the notation introduced in Fig. 1: There are 2 degrees for the angles ($\varphi_{12},\ \varphi_{13}$) of two legs of equal length (l), one degree of freedom for the angle φ_{14} (or φ_{24}) describing the angular position of a decorator (an upper body), and one degree for indicating the position of the hip (or for the position of one of the leg's tip B_i), e.g., x_H. This system undergoes a translation (it walks) on the right due to a synergic effect of (i) the torque between links 2 and 3 applied in a specific time instant, and (ii) periodic (pendulating) movement of link 4. A detailed description follows.

2.1 Governing equations

For hybrid dynamical systems [5], the equation of motion has to describe the interaction between the continuous and discrete parts of the corresponding mathematical model. Next, we set up the stepladder model with a decorator (SMWD) for both the continuous swing phase of the leg motion and the impact model, which has to be applied when both legs touch the ground.

First, the swing phase of the motion. is obtained from the usual Lagrangian approach. Let be the stance leg tip, e.g., (B_3) fixed at the origin, then the state of SMWD can be described by 6 variables: $\varphi_{13}, \varphi_{23}, \varphi_{14}, \dot{\varphi}_{13}, \dot{\varphi}_{23}, \dot{\varphi}_{14}$. Using $q = [\varphi_{13}, \varphi_{23}, \varphi_{14}]^T$, we can write the continuous dynamics as

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{u}, \quad (1)$$

where $\mathbf{D}(\mathbf{q})$ is the inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ contains Coriolis and centrifugal terms, $\mathbf{G}(\mathbf{q})$ contains gravity terms, $\mathbf{u}$ stands for the vector of active forces.

Second, the swing leg collision. It is an instantaneous change of velocity caused by an impulsive force at the leg tip that brings it to rest at the same time transmitting the momentum to the previous stance leg, yielding an expression

$$\dot{q}^+ = S_{col}\dot{q}^-, \quad (2)$$

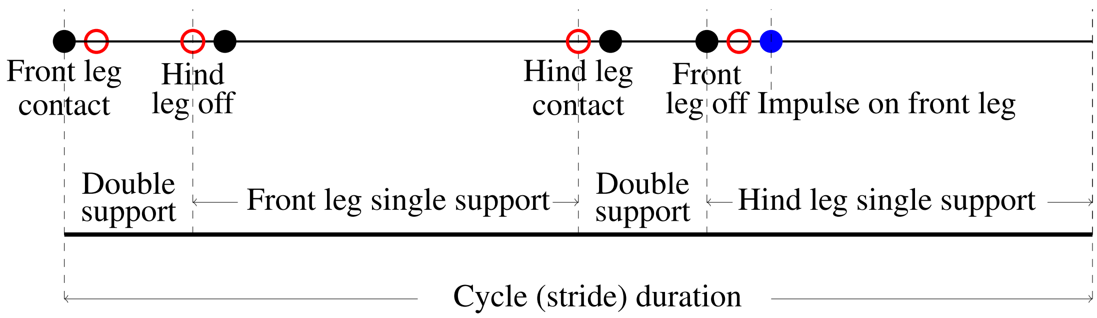

Fig. 2. Description of the individual phases of the gait cycle. A filled black circle stands for a stance leg, while an empty circle describes the leg that is either gaining or losing contact. The blue circle indicates the instant when the impulse on the front leg is applied

where the velocities just before ($\dot{q}^-$) and just after ($\dot{q}^+$) the impact, is governed by the impact model, see [2] for the collision operator S_{col} and other details.

2.2 Gait phases

Let a stable walking cycle starts with both feet on the ground, see Fig. 2. Let the relative angle $\varphi_{23} = 2\gamma$. The hind leg has a velocity away from the floor. During a step, the stance (front) leg is modeled as a hinge, connected to the floor. The swing leg moves freely as the other end of a double pendulum. This swing phase of the hind leg finishes when the leg tip B_2 reaches floor level (regarded as heel-strike). Let the relative angle $\varphi_{23} = 2\beta$ and $\gamma > \beta$. The swing leg makes a fully inelastic collision and becomes the new stance leg. Instantaneously, the former stance leg looses ground contact, and a new step begins. To impose the overall SLWD translation on the right, an impulsive force (momentum) is applied to the front swing leg causing an increment of the angular velocity of the front leg. A similar operator to (2) is proposed $\dot{q}^+ = S_{imp}\dot{q}^-$. Only then, the front leg swing phase begins. Similarly to the previous swing phase, the stance (hind) leg is modeled as a hinge, connected to the floor. The front swing leg moves freely as the other end of a double pendulum until it (tip B_3) reaches floor level. The relative angle is $\varphi_{23} = 2\gamma$, the cycle ends, and the length of the one-cycle step is $2(\sin\gamma - \sin\beta)$.

2.3 Limit cycle analysis

Striving for a (stable/continuous) cyclic walking, the initial conditions have to be the same as the conditions at the end of one cycle. Therefore, the consecutive phases of one gait cycle must be ensembled. Let us define a step-to-step function: $v_{n+1} = S_i(v_n)$, where $v = [q, \dot{q}]$ is the augmented state vector. Given the pendulating motion of link 4 is imposed, the simulation of one cycle comprises 5 phases (5 steps) and 5 operators S_i must be concatenated: (1) a smooth swing motion of the hind leg, (2) an abrupt collision at the heel strike, (3) the momentum transfer by the hip torque actuator, (4) a smooth swing motion of the front leg, and (5) an abrupt collision at the front leg heel. A walking cycle is thus specified by the requirement that the vector of initial conditions v_n results in identical initial conditions for the k_{th} subsequent step:

$$v_{n+k} = v_n, \quad v_{n+k} = S_i^k(v_n), \tag{3}$$

here, it is expected $k = 5$ or the multiples of 5 by an integer number. Because of the complexity of an analytical solution, a common practice is to employ the iteration procedure, which is regarded as a cycle-to-cycle function $v_5 = S_i^5(v_0)$, where v_0 is the initial guess. Monitoring the state of the system v once per cycle is known as Poincaré mapping. The gait is in a limit cycle if the corresponding initial conditions represent a fixed point on the Poincaré map. The implementation is made in the computer algebra system *Mathematica.*

3. Conclusion

In this work, we formulated a model for stable cyclic walking for the stepladder with an operator. It integrates the continuous and discrete parts of the overall one-cycle model and proposes a method for identifying the basin of attraction for stable walking. Now, encouraged by the successful implementation of the model, we are open to running a study on the stability of cyclic walking for different values of model parameters and the operator movement. Here, the expected result is finding an optimal stepladder walking regime (e.g., minimizing the energy input).

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