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REGULAR ARTICLE

# Nonparametric tests for serial independence in linear model against a possible autoregression of error terms

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## Abstract

When time series data follow a linear model, the innovations are often assumed to be serially independent. However, many time series also frequently display an autoregression of error terms. When testing a hypothesis on regression parameters, the tests can be distorted by a possible autoregression. Noting that we construct a class of non-parametric tests for the hypothesis of serial independence of error terms in the linear model against an alternative of linear autoregression. The main tool of the test criteria is the regression rank scores corresponding to the hypothetical model. The remarkable performance of the proposed tests is demonstrated by a simulation study and two real data examples.

**Keywords** Autoregression rank scores · Hypothesis testing · Linear regression · Rank test · Regression rank scores

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## 1 Introduction

When time series data follow a linear model, the error terms are often assumed to be serially independent. However, many time series often display an autoregression along with regression on nonrandom covariates. This problem is considered in a recent study by Wei et al. (2022), Alpuim and El-Shaarawi (2008), Basu and Michailidis (2015) and they construct a new unified test based on the empirical likelihood method for checking the AR error structure in autoregressive models. When the regression parameters are our main interest, then the possible autoregression of error terms distorts the conclusions of the used tests.

An autoregression can indicate a possible increasing trend, even if it does not exist (see Yu et al. 2002; Yue and Pilon 2003). Such a situation can be solved with two possible outcomes:

(i) We can start with a preliminary test of the serial independence against a possible autoregression of error terms, considering the linear model as a nuisance. Then, we can continue with inference on parameters of the linear model, depending on the conclusion of the test.

(ii) We can try developing the tests on parameters of the linear model, insensitive to a possible autoregression of error terms.

The tests of significance for parameters in the linear model, invariant to the autoregression of error terms [group (ii)], were recently constructed in Jurečková et al. (2023). The test criteria were based on the autoregression rank scores of the model under the null hypothesis.

Our ultimate aim is a construction of preliminary tests of the hypothesis of serial independence mentioned in group (i). We shall construct a family of nonparametric tests for the alternative autoregression of innovations of a time series, while it is known that the data themselves follow a linear model. The test criteria are based on the regression rank scores of the linear model under the null hypothesis of serial independence. The tests are nonparametric, thus independent on the probability distribution of the data. If such a test confirms a possible autoregression of error terms, our conclusions on the parameters of the linear model should be taken with caution.

The paper is organized as follows. The statement of the model and the general problem of testing the hypothesis of serial independence of error terms in the linear model against an alternative of linear autoregression is addressed in Sect. 2. In Sect. 3, we propose a family of tests for the hypothesis of serial independence of error terms in the linear model, based on regression rank scores. We analyze the asymptotic distribution of the test criterion under the null hypothesis, as well as under local autoregression alternatives. Monte Carlo simulations and real data analysis are presented in Sects. 4 and 5, respectively. Our concluding remarks are presented in Sect. 6.

## 2 Statement of the model

Our model is a time series of observations that follows a linear model and may include autoregressive error components:

$$\begin{aligned} y_t &= \beta_0 + \mathbf{x}_t^\top \boldsymbol{\beta} + e_t \\ &= \beta_0 + x_{t1}\beta_1 + ... + x_{tp}\beta_p + e_t \\ &= \beta_0 + \mathbf{x}_t^\top \boldsymbol{\beta} + \varphi_1 e_{t-1} + \ldots + \varphi_q e_{t-q} + u_t, \end{aligned} \tag{1}$$

for $t = 1, 2, \ldots, n$. Here the observed series are

$$y_{-q+1}, \ldots, y_0, y_1, \ldots, y_n \text{ of length } (n+q), \tag{2}$$

the regressors are $\mathbf{x}_t = \left(x_{t1}, \ldots, x_{tp}\right)^\top$ for $t = -q+1, \ldots, n$, and $\beta_0, \boldsymbol{\beta} = \left(\beta_1, \ldots, \beta_p\right)^\top$ are unknown regression parameters. For the convenience, we shall also denote

$$\begin{aligned} \boldsymbol{\beta}^* &= \left(\beta_0, \beta_1, \ldots, \beta_p\right)^\top, \\ \mathbf{x}_t^* &= \left(x_{t0}, x_{t1}, \ldots, x_{tp}\right)^\top; \quad x_{t0} = 1. \end{aligned} \tag{3}$$

Moreover, $\boldsymbol{\varphi} = \left(\varphi_1, \ldots, \varphi_q\right)^\top$ in (1) is an unknown autoregression parameter vector. Our hypothesis $\mathbf{H}_0$ states that $e_t,\ t = 1, \ldots, n$ are serially independent, identically distributed (i.i.d.) with distribution function $F$, density $f$, generally unknown. However, the circumstances of the experiment suggest that the error terms $e_t$ can follow a linear autoregressive model of a fixed order $q$, i.e.

$$e_t = \varphi_1 e_{t-1} + \ldots + \varphi_q e_{t-q} + u_t, \tag{4}$$

with autoregression parameters $\boldsymbol{\varphi}$, where the innovations $u_t$ in (4) generate a sequence of i.i.d. random variables with distribution function $F$ and innovation density $f$. We assume that $F$ and $f$ are generally unknown; only that $f$ belongs to a family $\mathcal{F}$ of densities satisfying

$$\int_{-\infty}^{\infty} x \mathrm{d}F(x) = 0 \quad , \qquad 0 < \int_{-\infty}^{\infty} x^2 \mathrm{d}F(x) = \sigma^2 < \infty. \tag{5}$$

Alpuim and El-Shaarawi (2008) studied the least-squares (LS) estimate of $(\beta_0, \boldsymbol{\beta})$ under (1), (4), as well as the maximum likelihood estimate under the normal distribution of $u_t$. Tuaç et al. (2018) studied the conditional maximum likelihood estimate of $(\beta_0, \boldsymbol{\beta})$, including possible asymmetric and heavy-tailed distributions of the $t$-type.

For the sake of simplicity, (4) is often written in the form

$$u_t = \Phi(B)\, e_t = e_t - \varphi_1 e_{t-1} - \ldots - \varphi_q e_{t-q}, \tag{6}$$

where $\Phi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \ldots - \varphi_q B^q$, and $B$ is called the backshift operator.

The problem is to verify the possible autoregression in the model (1). More precisely, we want to test the hypothesis of zero autoregression against an alternative of a local autoregression. Hence, we want to construct the tests of the following hypothesis on the parameters of model (1):

$$\mathbf{H}_0: \ \boldsymbol{\varphi} = \left(\varphi_1, \ldots, \varphi_q\right)^\top = \mathbf{0} \ \text{ with } q \text{ fixed, under } \boldsymbol{\beta}^* \neq \mathbf{0} \quad \text{unspecified.} \tag{7}$$

We propose a class of tests of $\mathbf{H}_0$ based on the *regression rank scores* corresponding to observations vectors $\widetilde{\mathbf{y}}_n = (y_1, \ldots, y_n)^\top$ under the assumed validity of $\mathbf{H}_0$. The tests are nonparametric in nature; however, the optimal test criterion contains the unobservable error terms $e_t$ as the weights. Hence, the error terms should be estimated from the residuals $y_t - \beta_0 - \mathbf{x}_t^\top \widehat{\boldsymbol{\beta}}$ with estimates $\hat{\beta}_0$ and $\widehat{\boldsymbol{\beta}}$ of the intercept $\beta_0$ and slopes vector $\boldsymbol{\beta}$, calculated under $\mathbf{H}_0$. For $F$ satisfying conditions given in (5) and under conditions on the covariates (X1)-(X3) given in Sect. 3, we can estimate $\boldsymbol{\beta}^*$ with the LS estimator based on observations (2). The LS estimator can be replaced with another suitable estimator.

## 3 Rank tests for $H_0$

Consider the hypothesis of serial independence in model (1), expressed as

$$\mathbf{H}_0: \ \boldsymbol{\varphi} = \mathbf{0}, \qquad \beta_0, \ \boldsymbol{\beta} \text{ unspecified.}$$

Under $\mathbf{H}_0$, the observations follow the ordinary regression model

$$y_t = \mathbf{x}_t^{*\top} \boldsymbol{\beta}^* + e_t = \beta_0 + x_{t1}\beta_1 + \ldots + x_{tp}\beta_p + e_t, \quad t = 1, \ldots, n \tag{8}$$

with i.i.d. error terms $e_t$. We want to verify the hypothesis $\mathbf{H}_0$ against the local autoregression alternative $y_t = \mathbf{x}_t^{*\top} \boldsymbol{\beta}^* + \varphi_1 e_{t-1} + \ldots + \varphi_q e_{t-q} + u_t, \quad t = 1, \ldots, n$ with $\boldsymbol{\varphi} = \left(\varphi_1, \ldots, \varphi_q\right)^\top$ satisfying

$$\mathbf{K}_n: \ \boldsymbol{\varphi} = \boldsymbol{\varphi}_n = n^{-1/2} \boldsymbol{\varphi}^*, \ \text{ with } \ \boldsymbol{\varphi}^* \in \mathbb{R}, \quad q \text{is fixed}, \ \ \boldsymbol{\beta}^* \text{ unspecified.} \tag{9}$$

We propose a nonparametric test based on *regression rank scores* corresponding to observations $\widetilde{\mathbf{y}}_n = (y_1, \ldots, y_n)^\top$ under the validity of $\mathbf{H}_0$.

The regression rank scores $\widehat{\mathbf{a}}_n(\alpha) = \left(\hat{a}_{n1}(\alpha), \ldots, \hat{a}_{nn}(\alpha)\right)^\top, \ 0 \leq \alpha \leq 1$ corresponding to model (1) under hypothesis $\mathbf{H}_0$ are defined as the vector of solutions of the linear programming problem

$$\begin{cases} max \sum_{t=1}^{n} y_t \hat{a}_{nt}(\alpha) & \\ \sum_{t=1}^{n} (\hat{a}_{nt}(\alpha) - (1-\alpha)) & = 0 \\ \sum_{t=1}^{n-1} x_{tj} (\hat{a}_{nt}(\alpha) - (1-\alpha)) & = 0, \ \ j = 1, \ldots, p \\ \widehat{\mathbf{a}}_n(\alpha) \in [0, 1]^n, \quad 0 \leq \alpha \leq 1. & \end{cases} \tag{10}$$

The regression rank scores are *regression-invariant*. Indeed, denoting the $n \times (p+1)$ matrix

$$\mathbf{X}_n^* = \begin{bmatrix} \mathbf{x}_1^{*\top} \\ \dots \\ \mathbf{x}_n^{*\top} \end{bmatrix} = \begin{bmatrix} 1, \mathbf{x}_1^\top \\ \dots \\ 1, \mathbf{x}_n^\top \end{bmatrix},$$

we see that (10) implies that $\widehat{\mathbf{a}}_n(\alpha)$ can also be formally written as a solution of the linear program

$$\begin{cases} max \sum_{t=1}^n e_t \hat{a}_{nt}(\alpha) \\ \mathbf{X}_n^{*\top} \hat{\mathbf{a}}_n(\alpha) = (1-\alpha)\mathbf{X}_n^{*\top} \mathbf{1}_n \\ \widehat{\mathbf{a}}_n(\alpha) \in [0, 1]^n, \quad 0 \le \alpha \le 1. \end{cases} \tag{11}$$

We shall construct a family of tests of the hypothesis $\mathbf{H}_0$ for the model (1), based on regression rank scores and analyze the asymptotic distribution of the test criterion under the null hypothesis as well as under local (contiguous) autoregression alternatives. The tests will function for the (unknown) densities $f$ of $u_t$ belonging to the family $\mathcal{F}$ of exponentially tailed densities, satisfying (5) and the following conditions on the tails:

**(F1)** $f$ is positive and absolutely continuous, with a.e. derivative $f'$ and finite Fisher information $\mathcal{I}(f) = \int \left(\frac{f'(x)}{f(x)}\right)^2 f(x)\mathrm{d}x < \infty$; moreover, there exists $K_f \ge 0$ such that $f$ has two bounded derivatives $f'$ and $f''$ for all $|x| > K_f$;

**(F2)** $f$ is monotonically decreasing to 0 as $x \to \pm\infty$ and

$$\lim_{x \to -\infty} \frac{-\log F(x)}{b|x|^r} = \lim_{x \to \infty} \frac{-\log(1 - F(x))}{b|x|^r} = 1$$

for some $b > 0$ and $r \ge 1$.

Moreover, we impose the following conditions on the regression matrix $\mathbf{X}_n$ :

**(X1)** The matrix $\mathbf{Q}_n = n^{-1}\mathbf{X}_n^\top \mathbf{X}_n$ is positive definite of order $p$ for $n \ge n_0$;

**(X2)** $n^{-1} \sum_{t=1}^n \|\mathbf{x}_{nt}\|^4 = O(1)$ as $n \to \infty$;

**(X3)** $\lim_{n\to\infty} \max_{1 \le t \le n} \left\{ n^{-1} \mathbf{x}_{nt}^\top \mathbf{Q}_n^{-1} \mathbf{x}_{nt} \right\} = \mathbf{0}$.

The regression rank scores tests were developed in Gutenbrunner and Jurečková (1992) and Gutenbrunner et al. (1993); they extend the ordinary rank tests to the linear model. Similarly as in the ordinary rank tests, in the test criteria, the vectors $\hat{\mathbf{a}}_n(\alpha)$ are weighted with a non-decreasing, square-integrable score function $J : (0, 1) \to \mathbb{R}$, such that $J(1-u) = -J(u),\ 0 < u < 1,\ \int_0^1 J^2(u)\mathrm{d}u = A_J^2 < \infty$. $J'(u)$ exists for $u \in (0, \alpha_0) \cup (1 - \alpha_0, 1)$ and, in this domain, and satisfies the following Chernoff-Savage condition.

$$|J'(u)| \le c\,(u(1-u))^{-1-\delta},\ 0 < \delta < \frac{1}{4}. \tag{12}$$

The test is based on the vector of scores $\tilde{J}_n = \left(\tilde{J}_{n;1}, \ldots, \tilde{J}_{n;n}\right)^\top$ with

$$\tilde{J}_{n,t} = -\int_0^1 J(u)\, \mathrm{d}\hat{a}_{n,t}(u), \quad t = 1, \ldots, n. \tag{13}$$

Efficient algorithms for the computation of such scores can be found in Koenker and d'Orey (1987, 1994). The rank tests, most typical in the practice, are based on the scores generated by the following functions $J$:

(i) The Wilcoxon scores with $J(u) = u - \frac{1}{2}$, efficient under the logistic $F$, but suitable in many situations;
(ii) The Laplace or median test scores with $J(u) = \text{sign}\left(u - \frac{1}{2}\right)$. They are efficient under double exponential innovations and recommended under heavy-tailed distributions;
(iii) The van der Waerden or normal scores $J(u) = F^{-1}(u)$ (where $F$ stands for the standard normal distribution function), which are efficient under Gaussian innovations.

The test is based on *the vector of linear regression rank score statistics*

$$\begin{aligned} \mathbf{S}_{J;n} &= \left(S_{n1}, \ldots, S_{nq}\right)^\top, \\ S_{nj} &= n^{-\frac{1}{2}} \sum_{t=1}^{n} \hat{e}_{t-j}\, \tilde{J}_{n,t}, \ \ j = 1, \ldots, q, \end{aligned} \tag{14}$$

where

$$\hat{e}_t = y_t - \mathbf{x}_t^{*\top} \widehat{\boldsymbol{\beta}}_n^*, \ \ t = -q+1, \ldots, n-1 \tag{15}$$

are estimates of unobservable terms $y_t - \mathbf{x}_t^{*\top}\boldsymbol{\beta}^*$ with $\boldsymbol{\beta}^*$ replaced with a suitable estimate $\widehat{\boldsymbol{\beta}}_n^*$, $t = -q+1, \ldots, n-1$. Because the distributions satisfying (F1)-(F2) have finite second moments, we can use the LS estimator of $\boldsymbol{\beta}^*$. Indeed,

$$\hat{e}_t = y_t - \mathbf{x}_t^{*\top} \widehat{\boldsymbol{\beta}}_n^* = y_t - \mathbf{x}_t^{*\top} (\mathbf{X}^{*\top}\mathbf{X}^*)^{-1} \mathbf{X}^{*\top}\mathbf{Y} = e_t - \mathbf{h}_t^{*\top} e_t \tag{16}$$

where $\mathbf{h}_t^{*\top}$ is the $t$-th row of the projection matrix $\mathbf{H}_n^* = \mathbf{X}^*(\mathbf{X}^{*\top}\mathbf{X}^*)^{-1}\mathbf{X}^{*\top}$. Hence, $\hat{\mathbf{e}} = (\mathbf{I}_n - \mathbf{H}_n^*)\mathbf{e}$ is the orthogonal complement of $\mathbf{e}$ on the space spanned by the columns of $\mathbf{X}_n^*$.

However, for non-normal distributions, we recommend using the $\alpha$-trimmed LS estimator of $\boldsymbol{\beta}^*$, $(0 < \alpha < 1/2)$, constructed in Koenker and Bassett Jr (1978) and studied in Ruppert and Carroll (1980). It is the weighted least squares estimator $\mathbf{T}_n(\alpha)$ with the weights

$$c_t = c_{nt} = \hat{a}_{nt}(\alpha) - \hat{a}_{nt}(1-\alpha), \ \ t = 1, \ldots, n. \tag{17}$$

More explicitly,

$$\mathbf{T}_n(\alpha) = (\mathbf{X}_n^{*\top} \mathbf{C}_n \mathbf{X}_n^*)^{-1} \mathbf{X}_n^{*\top} \mathbf{C}_n \mathbf{Y}_n \tag{18}$$

where $\mathbf{C}_n = diag(c_t)$ is the diagonal matrix with diagonal $(c_1, \ldots, c_n)$. If the inverse in (18) does not exist, it is replaced by a generalized inverse. Under the above conditions, $n^{1/2} \left( \mathbf{T}_n(\alpha) - \boldsymbol{\beta}^* - \mathbf{e}_1 \delta(\alpha) \right)$ is asymptotically normal with zero expectation and covariance matrix $(1 - 2\alpha)^{-1} \mathbf{Q} \sigma^2(\alpha, F)$, where $\mathbf{e}_1 = (1, 0, \ldots, 0)^\top \in \mathbb{R}_{p+1}$ and $\delta(\alpha) = (1 - 2\alpha)^{-1} \int_\alpha^{1-\alpha} F^{-1}(u)du < \infty$. Under $F$ symmetric, $\mathbf{T}_n(\alpha)$ is an asymptotically unbiased estimator of $\boldsymbol{\beta}^*$.

The test criterion is a specific quadratic form of $\mathbf{S}_{J;n}$. The asymptotic behavior of statistics (14) is analyzed in Gutenbrunner and Jurečková (1992) and Gutenbrunner et al. (1993). Koul and Saleh (1995) introduced the autoregression rank scores and the tests based on them were developed in Hallin and Jurecková (1999) and Hallin et al. (2007), among others. For applications of these tests in climatology, we refer e.g. to Hallin et al. (1997) and Hallin et al. (1999).

Summarizing, as the test criterion for $\mathbf{H}_0$ against the linear autoregression of order $q$ we propose the quadratic form

$$T_{J;n} = n A_J^{-2} \mathbf{S}_{J;n}^\top \left( \sum_{t=0}^{n-1} \hat{\mathbf{e}}_t \hat{\mathbf{e}}_t^\top \right)^{-1} \mathbf{S}_{J;n}, \quad A_J^2 = \int_0^1 J^2(u) du \tag{19}$$

where

$$\hat{\mathbf{e}}_t = (\hat{e}_t, \hat{e}_{t-1}, \ldots \hat{e}_{t-q+1})^\top, \; t = 0, \ldots, n-1. \tag{20}$$

Inserting the R-estimate $\widehat{\boldsymbol{\beta}}_{nq}$ in (14) and (20), we obtain the asymptotic central $\chi^2$ distribution of $T_{J;n}$ under $\mathbf{H}_0$. Under the local (Pitman) alternative $\mathbf{K}_n$, we get the asymptotic noncentral $\chi^2$ distribution, in view of (5). More precisely, we can state the following theorem.

**Theorem 1** *Assume that the observations follow the model (1) with the parent distribution of model errors satisfying conditions (F1) and (F2) and the regression matrix* $\mathbf{X}$ *satisfying (X1)-(X3). Then*

*(i) Under the hypothesis,* $\mathbf{H}_0$, *the criterion* $T_{J;n}$ *has asymptotically the central* $\chi^2$ *distribution with q degrees of freedom.*

*(ii) Under the local alternative,* $\mathbf{K}_n$, *the asymptotic distribution of the criterion* $T_{J;n}$ *is the non-central* $\chi^2$ *distribution with q degrees of freedom and non-centrality parameter*

$$\eta^2 = \frac{\gamma^2(J, F)}{A_J^2} \sigma^2 \boldsymbol{\varphi}^{*\top} \boldsymbol{\varphi}^*, \textit{ where}$$
$$\gamma(J, F) = -\int_0^1 J(u) \mathrm{d} f(F^{-1}(u)). \tag{21}$$

The proof of the theorem is omitted, because it follows the lines of the proofs of analogous results in Gutenbrunner and Jurečková (1992) and Gutenbrunner et al. (1993).

## 4 Computation and simulation study

In order to evaluate the performance of the proposed testing procedure, a simulation study and a real data application have been provided. In the simulation study, we take a simple linear regression model with only one covariate and an AR(1) error structure. That is, we consider the following linear regression model

$$y_t = \beta_0 + x_{t1}\beta_1 + e_t, \tag{22}$$

$$e_t = \phi_0 - \phi_1 e_{t-1} + u_t, t = 1, 2, \ldots, n. \tag{23}$$

*Simulation Settings.* The regression covariate $\mathbf{x}_{t1}$ is generated from $\mathbf{U}(0, 1)$. Both intercepts $\boldsymbol{\beta}_0$, and $\boldsymbol{\phi}_0$ are set to 0, and the regression parameter $\boldsymbol{\beta}_1$ is set to 3. We should note that different values of the regression parameter have been used in the simulation study, and it has been observed that the value of the regression coefficient does not cause any significant difference in the power of the test. We used Wilcoxon, Median, and Van der Waerden scores to compute the test statistics and compare their performances. The null hypothesis $\mathbf{H}_0 : \phi_1 = 0$ is rejected if the calculated test score is higher than the central value $\chi^2$ with q degrees of freedom and level 5% in each case. To compute the regression rank scores, we employed the “quantreg” package (Koenker (2021)) for R. Throughout this simulation study, for each experiment we ran 10,000 replications. The rejected cases in each run are counted, and the calculated powers of the test are given in the tables for different error distribution assumptions. The empirical power of the proposed test is compared for different values of autoregressive parameters given as follows: $\boldsymbol{\phi} = -0.9, -0.5, -0.3, -0.1, 0, 0.1, 0.3, 0.5, 0.9$

To illustrate the behavior of the proposed method, we used different distribution assumptions for innovations $\mathbf{u}_t$. We considered symmetric and asymmetric distributions to see the strength of the proposed test under different error distribution structures. We used the following symmetric distributions.

(i) $\mathbf{u}_t \sim N(0, 1)$,
(ii) $\mathbf{u}_t \sim t(3)$,
(iii) $\mathbf{u}_t \sim Cauchy(0, 1)$,
(iv) $\mathbf{u}_t \sim Laplace(0, 1)$.

The asymmetric distributions are Azzalini-type skew-normal (Azzalini 1985, 1986) $\mathbf{u}_t \sim sn(0, 1, \lambda = 0.5)$ and Azzalini type skew-t (Azzalini 2005) $\mathbf{u}_t \sim st_3(0, 1, \lambda = 0.5)$ with the sample sizes $n = 20, 100, 300, 500$. To generate random samples from the distributions in R, we used the "VGAM" package (Yee 2010) for Laplace distribution and the "sn" package (Azzalini 2023) for Azzalini-type skew-normal and Azzalini-type skew-t distributions. Note that in this simulation study, the degrees of freedom ($\nu$) of the $t$ and the skew$t$ distributions are taken as fixed ($\nu = 3$). Throughout the simulation study and real data examples, R statistical software (Team 2024) is used.

*Simulation results.* Table 1 summarizes the calculated powers of the test when the error term has different symmetric distributions. First, it is easily seen that when the sample size gets larger, the proposed test results get better in every scenario. Considering the score functions, even if there are slight differences between the results of three scores for the small sample size, the results from all the score functions are satisfactory for the proposed hypothesis test. It can be easily seen from Table 1 that if the autoregression parameter takes a value close to zero, which means the autoregression parameter is insignificant, the power of the test reduces drastically, which is an expected result. Also, the performance of the proposed technique is not affected when the distribution assumption is changed with another symmetric distribution like t, Laplace, or even Cauchy. This result shows that the proposed method can be used in any distribution assumption. In the second case of the simulation, the performance of the test is investigated under the assumption of asymmetric error distribution. Table 2 shows the power of the test results under the assumptions of Azzalini-type asymmetric distribution. Azzalini-type skew distributions are flexible distributions because they have skew-symmetric properties. The results of the skew distributions are also promising. The power of the test has nearly the same performance as the symmetric case. If we compare the performance of the scores used in each distribution, the Wilcoxon score gave more accurate results even with small sample sizes compared to the other two scores. Under the symmetric thick-tailed assumption, the Median score showed superior success compared to the other two scores. In the cases of Cauchy and Laplace distribution assumptions, as in the normal distribution, the Wilcoxon score exhibited better performance than the other two scores. For the skew-distributed assumptions, the Wilcoxon and Median scores showed comparable performance under the skew-normal assumption, while the Wilcoxon score outperformed the other two scores in the skew-t distribution assumption.

In summary, the proposed test method for the autoregressive error term regression model is effective under various sample sizes and distribution-free even for asymmetric cases. The proposed test based on all the considered scores is powerful for rejecting the null hypothesis, while the error term has an autoregressive structure.

## 5 Real data examples

In this section, we provide two real-data examples to illustrate the performance of the proposed method in the application.

The first real data set consists of the number of formal education students and the housing capacity from the Turkish Higher Education Loans and Dormitories Institution. The data contain an annual basis between 1992-2021 with a sample size $n = 30$ and available online at National Education Statistics, Formal Education 2020/'21 report (T.C.MEB 2021).

First, we consider a linear regression model with a housing capacity $(y_t)$ as dependent and No. students $(x_t)$ as explanatory variables. Here, we assume that the distribution has a finite second moment and use the LS estimation method to

**Table 1** Powers of the test ($\tau_n$ with Wilcoxon, Median, and Van der Waerden scores) for various sample sizes, $\beta_1 = 3$ for different autoregressive values and symmetric error distributions

| $\phi_1$ | n | Normal Wilcoxon | Median | Waerden | $t_3$ Wilcoxon | Median | Waerden | Cauchy Wilcoxon | Median | Waerden | Laplace Wilcoxon | Median | Waerden |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| −0.9 | 20 | 0.9164 | 0.8521 | 0.6157 | 0.8351 | 0.9331 | 0.6312 | 0.9492 | 0.8943 | 0.6642 | 0.9236 | 0.8637 | 0.6331 |
| | 100 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9992 | 0.9990 | 1.0000 | 1.0000 | 1.0000 |
| | 300 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | 500 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| −0.5 | 20 | 0.4602 | 0.3614 | 0.1473 | 0.5345 | 0.4450 | 0.2200 | 0.7084 | 0.4610 | 0.3321 | 0.5358 | 0.4509 | 0.2063 |
| | 100 | 0.9999 | 0.9833 | 0.9947 | 1.0000 | 0.9963 | 0.9967 | 1.0000 | 0.9502 | 0.9592 | 0.9999 | 0.9967 | 0.9978 |
| | 300 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9982 | 0.9641 | 0.9788 | 1.0000 | 1.0000 | 1.0000 |
| | 500 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9748 | 0.9960 | 1.0000 | 1.0000 | 1.0000 |
| −0.3 | 20 | 0.1945 | 0.1715 | 0.0476 | 0.2401 | 0.2012 | 0.0763 | 0.4155 | 0.1937 | 0.1656 | 0.2212 | 0.2150 | 0.0728 |
| | 100 | 0.8313 | 0.6791 | 0.7643 | 0.9358 | 0.8521 | 0.8678 | 0.9961 | 0.8179 | 0.8611 | 0.9104 | 0.8752 | 0.8603 |
| | 300 | 0.9970 | 0.9875 | 0.9997 | 0.9735 | 0.9869 | 0.9986 | 1.0000 | 0.9008 | 0.9512 | 0.9994 | 0.9995 | 0.9645 |
| | 500 | 0.9980 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9077 | 0.9628 | 1.0000 | 1.0000 | 1.0000 |

**Table 1** continued

| $\phi_1$ | n | Normal Wilcoxon | Median | Waerden | $t_3$ Wilcoxon | Median | Waerden | Cauchy Wilcoxon | Median | Waerden | Laplace Wilcoxon | Median | Waerden |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| −0.1 | 20 | 0.0610 | 0.0674 | 0.0130 | 0.0652 | 0.0638 | 0.0152 | 0.0886 | 0.0482 | 0.0341 | 0.0529 | 0.0755 | 0.0187 |
| | 100 | 0.1784 | 0.1378 | 0.1309 | 0.2221 | 0.1981 | 0.1969 | 0.7812 | 0.3797 | 0.6921 | 0.2448 | 0.2325 | 0.1754 |
| | 300 | 0.4301 | 0.2866 | 0.3826 | 0.6065 | 0.2947 | 0.3883 | 0.9761 | 0.7286 | 0.8947 | 0.5534 | 0.5719 | 0.4802 |
| | 500 | 0.5960 | 0.4413 | 0.5124 | 0.8267 | 0.7416 | 0.5215 | 0.9873 | 0.7971 | 0.9117 | 0.7662 | 0.8021 | 0.6952 |
| 0.0 | 20 | 0.0352 | 0.0509 | 0.0088 | 0.0279 | 0.0426 | 0.0093 | 0.0224 | 0.0178 | 0.0078 | 0.0318 | 0.0417 | 0.0063 |
| | 100 | 0.0551 | 0.0484 | 0.0343 | 0.0438 | 0.0465 | 0.0352 | 0.0295 | 0.0180 | 0.0331 | 0.0457 | 0.0476 | 0.0351 |
| | 300 | 0.0592 | 0.0479 | 0.0426 | 0.0510 | 0.0509 | 0.0421 | 0.0326 | 0.0216 | 0.0439 | 0.0511 | 0.0467 | 0.0399 |
| | 500 | 0.0501 | 0.0493 | 0.0479 | 0.0500 | 0.0665 | 0.0418 | 0.0451 | 0.0213 | 0.0460 | 0.0473 | 0.0529 | 0.0420 |
| 0.1 | 20 | 0.0369 | 0.0463 | 0.0057 | 0.0338 | 0.0432 | 0.0065 | 0.0229 | 0.0253 | 0.0074 | 0.0227 | 0.0456 | 0.0058 |
| | 100 | 0.1137 | 0.1016 | 0.0941 | 0.1705 | 0.1620 | 0.1443 | 0.7402 | 0.3507 | 0.7425 | 0.1885 | 0.1908 | 0.1229 |
| | 300 | 0.3712 | 0.2639 | 0.3421 | 0.5837 | 0.2570 | 0.3341 | 0.9727 | 0.7162 | 0.9723 | 0.5219 | 0.5491 | 0.4386 |
| | 500 | 0.5644 | 0.4150 | 0.4578 | 0.8239 | 0.7210 | 0.7012 | 0.9836 | 0.7911 | 0.9812 | 0.7614 | 0.7851 | 0.6754 |
| 0.3 | 20 | 0.0894 | 0.0903 | 0.0200 | 0.1145 | 0.1076 | 0.0264 | 0.1823 | 0.1145 | 0.0630 | 0.1098 | 0.1234 | 0.0314 |
| | 100 | 0.7912 | 0.6063 | 0.6998 | 0.9034 | 0.8121 | 0.8476 | 0.9908 | 0.8165 | 0.9450 | 0.8800 | 0.8348 | 0.8158 |
| | 300 | 0.9956 | 0.9839 | 0.9980 | 0.9693 | 0.9837 | 0.9981 | 0.9999 | 0.8959 | 0.9783 | 0.9999 | 0.9999 | 0.9426 |
| | 500 | 0.9974 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9045 | 0.9846 | 1.0000 | 1.0000 | 1.0000 |
| 0.5 | 20 | 0.2407 | 0.2167 | 0.0686 | 0.3248 | 0.2641 | 0.1013 | 0.4472 | 0.3145 | 0.1856 | 0.2815 | 0.2747 | 0.0971 |
| | 100 | 0.9955 | 0.9665 | 0.9902 | 1.0000 | 0.9934 | 0.9961 | 1.0000 | 0.9443 | 0.9786 | 1.0000 | 0.9938 | 0.9980 |
| | 300 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9654 | 0.9845 | 1.0000 | 1.0000 | 1.0000 |
| | 500 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9773 | 0.9992 | 1.0000 | 1.0000 | 1.0000 |
| 0.9 | 20 | 0.7132 | 0.6415 | 0.3537 | 0.7319 | 0.6576 | 0.3631 | 0.7884 | 0.7303 | 0.4297 | 0.7407 | 0.6537 | 0.3616 |
| | 100 | 0.9989 | 0.9999 | 0.9997 | 1.0000 | 0.9999 | 1.0000 | 0.9999 | 0.9990 | 0.9991 | 1.0000 | 0.9999 | 0.9961 |
| | 300 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | 500 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

**Table 2** Powers of the test ($\tau_n$ with Wilcoxon, Median, and Van der Waerden scores) for various sample sizes, $\beta_1 = 3$ for different autoregressive values and asymmetric error distributions

| $\phi_1$ | n | Skew normal Wilcoxon | Median | Vaerden | Skew-t Wilcoxon | Median | Vaerden |
|---|---|---|---|---|---|---|---|
| −0.9 | 20 | 0.8947 | 0.8581 | 0.6160 | 0.9420 | 0.8682 | 0.6221 |
| | 100 | 1.0000 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | 300 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | 500 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| −0.5 | 20 | 0.4342 | 0.3652 | 0.1389 | 0.5342 | 0.4253 | 0.1727 |
| | 100 | 0.9999 | 0.9779 | 0.9927 | 1.0000 | 0.9976 | 0.9953 |
| | 300 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | 500 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| −0.3 | 20 | 0.2041 | 0.1669 | 0.0427 | 0.2655 | 0.2029 | 0.0636 |
| | 100 | 0.8385 | 0.6752 | 0.7584 | 0.9300 | 0.8540 | 0.8569 |
| | 300 | 0.9615 | 0.9859 | 0.9992 | 0.9521 | 0.9997 | 0.9995 |
| | 500 | 0.9871 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| −0.1 | 20 | 0.0551 | 0.0685 | 0.0115 | 0.0691 | 0.0649 | 0.0134 |
| | 100 | 0.1967 | 0.1389 | 0.1307 | 0.2733 | 0.1961 | 0.1966 |
| | 300 | 0.4220 | 0.2908 | 0.3913 | 0.6684 | 0.5076 | 0.5605 |
| | 500 | 0.6132 | 0.3561 | 0.5028 | 0.8212 | 0.6927 | 0.7811 |
| 0 | 20 | 0.0358 | 0.0518 | 0.0072 | 0.0356 | 0.0404 | 0.0063 |
| | 100 | 0.0475 | 0.0497 | 0.0336 | 0.0322 | 0.0435 | 0.0312 |
| | 300 | 0.0452 | 0.0530 | 0.0465 | 0.0591 | 0.0469 | 0.0448 |
| | 500 | 0.0500 | 0.0574 | 0.0487 | 0.0604 | 0.0475 | 0.0485 |
| 0.1 | 20 | 0.0347 | 0.0483 | 0.0054 | 0.0312 | 0.0419 | 0.0055 |
| | 100 | 0.1112 | 0.1021 | 0.0964 | 0.1897 | 0.1671 | 0.1566 |
| | 300 | 0.3775 | 0.2576 | 0.3291 | 0.6143 | 0.4932 | 0.5260 |
| | 500 | 0.5586 | 0.4125 | 0.5114 | 0.8195 | 0.7280 | 0.7547 |
| 0.3 | 20 | 0.0754 | 0.0924 | 0.0188 | 0.0711 | 0.1067 | 0.0237 |
| | 100 | 0.7697 | 0.6044 | 0.6921 | 0.9053 | 0.8108 | 0.8599 |
| | 300 | 0.9821 | 0.9822 | 0.9986 | 0.9999 | 0.9996 | 0.9999 |
| | 500 | 0.9941 | 0.9617 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 0.5 | 20 | 0.2462 | 0.2274 | 0.0643 | 0.2674 | 0.2665 | 0.0923 |
| | 100 | 0.9963 | 0.9683 | 0.9898 | 1.0000 | 0.9954 | 0.9965 |
| | 300 | 1.0000 | 0.9981 | 0.9997 | 1.0000 | 0.9999 | 1.0000 |
| | 500 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 0.9 | 20 | 0.8165 | 0.7461 | 0.4692 | 0.8323 | 0.7434 | 0.4665 |
| | 100 | 1.0000 | 1.0000 | 1.0000 | 0.9967 | 1.0000 | 1.0000 |
| | 300 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | 500 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

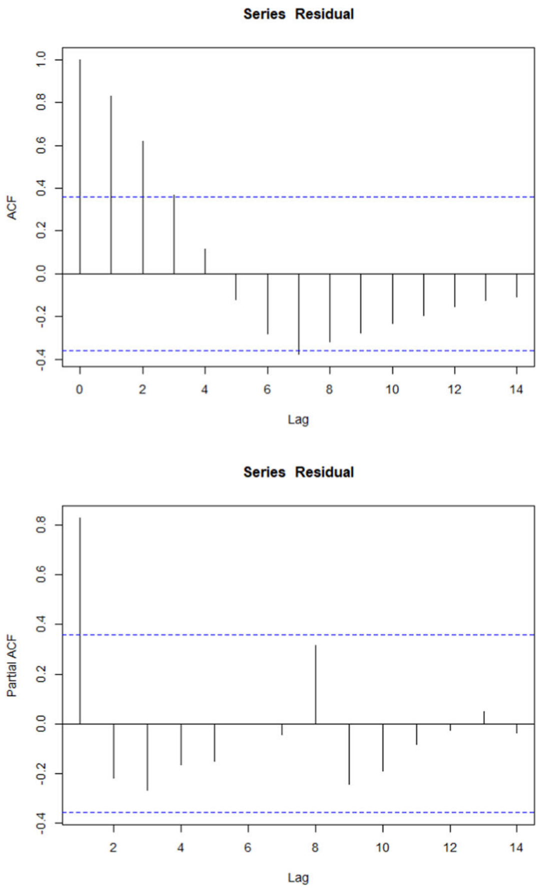


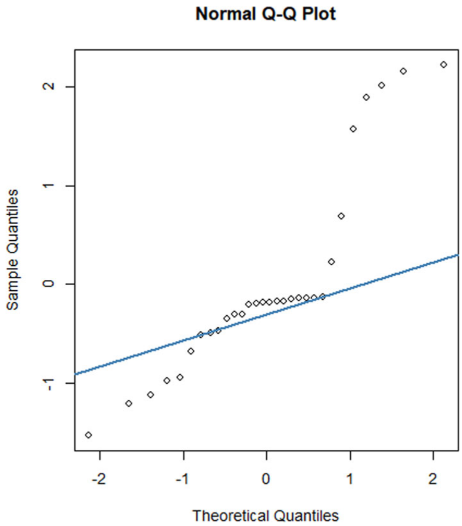

**Fig. 1** ACF and PACF of the residuals of the students and the housing capacity data

estimate the parameters. When the simple linear regression model is considered without an autoregressive error term, we obtain the intercept as insignificant and $\widehat{\beta}_1 = 0.1151$. We further examined the residuals in terms of error distribution and the AR structure. In Fig. 1, the autocorrelation function (ACF) and the partial autocorrelation function (PACF) plots of the residuals obtained from the LS estimates are presented. We observe from these plots that ACF decays exponentially to zero and PACF cuts off after lag 1. Therefore, these results suggest a significant AR(1) correlation structure. The result from the Durbin-Watson (DW) statistic of the data ($DW = 0.1993$, $p - value < 0.001$) also supports the presence of positive autocorrelation among residuals. "auto.arima" function in R package "forecast" (Hyndman and Khandakar 2008) is used to determine whether there is an ARIMA process on the data, and the function estimates an AR(1) structure in the residuals with $\widehat{\phi}_1 = 0.9226$.

$$\widehat{y}_t = 0.1151 x_t, \tag{24}$$

$$\widehat{e}_t = 0.9226 e_{t-1}, t = 1, 2, \ldots, 30. \tag{25}$$

**Fig. 2** Q–Q plot of the residuals of the students and the housing capacity data




Just as in the simulation study, we used Wilcoxon, Median, and Van der Waerden scores to compute the test statistics for the real data. We compute the scores, and the null hypothesis is rejected if the calculated test score is higher than the central $\chi^2$ value with q degrees of freedom and level 5%. Since all three scores reject the null hypothesis, we can conclude that the autoregressive parameter is significant for the housing capacity and number of students in these data according to the proposed method.

The Q–Q graph given in Fig. 2 was drawn to determine the distribution type of the residuals. According to this graph, it can be said that the data are light-tailed with right-skewed. The proposed method was able to accurately detect autocorrelation in real-life skewed data.

The second example, taken from the book by Kutner et al. (1983) is an economic case study featuring the "Blaisdell Company." The dataset contains two variables with a sample size of $n = 20$. We fit a simple linear regression model with comsales (company sales in \$ millions) as the response and indsales (industry sales in \$ millions) as the predictor. Similar to the previous example, we assume that the distribution has a finite second moment and the LS estimation method is employed to estimate the model parameters without considering any autocorrelation structure. By conducting further analysis on the residuals, focusing on the AR structure and the distribution in error terms, we first used ACF and the PACF plots. Figure 3 displays that ACF decays exponentially to zero and PACF cuts off after lag 1. These findings strongly suggest a significant AR(1) correlation structure in the residuals. Also, the result of the DW test indicates a significant positive autocorrelation among the residuals $(DW = 0.1839,\ p - value < 0.001)$. The result of DW statistics supports the interpretation of the ACF and PACF plots. The results obtained from the "auto.arima" function showed that the residuals had a first-order significant autocorrelation with a coefficient of 0.6052. According to these results, the proposed hypothesis tests were

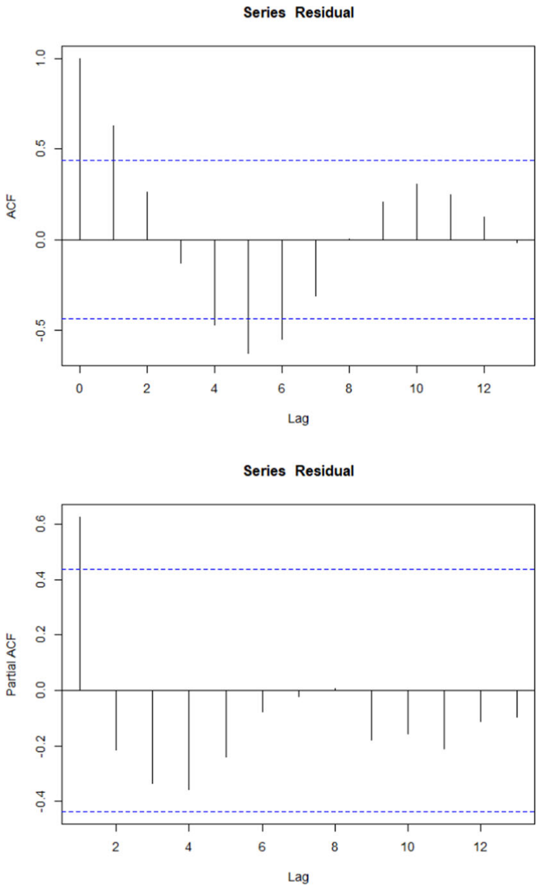



**Fig. 3** ACF and PCF of the residuals of the Blaisdell Company data

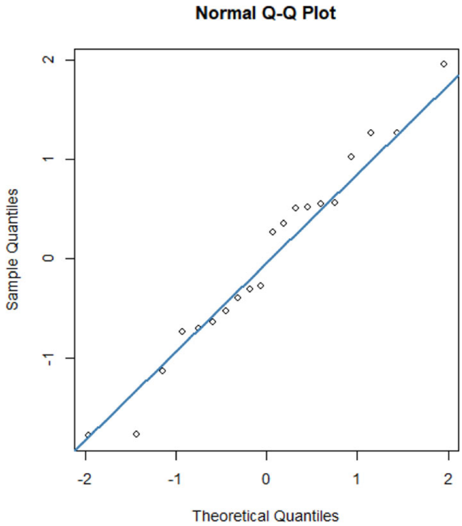
applied using the following fitted models.

$$\widehat{y}_t = -1.4547 + 0.1762x_t, \tag{26}$$

$$\widehat{e}_t = 0.6052e_{t-1}, t = 1, 2, \ldots, 20. \tag{27}$$

Wilcoxon, Median, and Van der Waerden scores are also calculated for the "Blaisdell Company" data. The null hypothesis is rejected by all three scores, leading to the conclusion that the autoregressive parameter is significant for the linear regression with the autocorrelated error term. The Q–Q graph generated for the "Blaisdell Company" data in Fig. 4 indicates an approximately normal distribution for the residuals. This indicates that, under normality assumptions, the proposed tests performed successfully as expected.

**Fig. 4** Q–Q plot of the residuals of the Blaisdell Company data




## 6 Conclusion

Our main goal was to verify the hypothesis of serial independence of model errors in the linear regression model, to prevent a possible distortion caused by a possible autoregression of model errors. We construct a class of non-parametric tests for hypothesis $\mathbf{H}_0$ in (7) against the local autoregression alternative of model errors. The construction of tests follows the general idea of non-parametric tests on selected components of the regression parameter, proposed in Gutenbrunner et al. (1993). The test criterion $T_{J;n}$ in (19) is based on the regression rank scores of the model (1) with estimated unobservable innovations. We reject $\mathbf{H}_0$ on the level $\alpha$ if

$$T_{J;n} \geq \chi^2_q(1-\alpha)$$

with $\chi^2_q(1-\alpha)$ is the $(1-\alpha)$ quantile of the $\chi^2$ distribution with $q$ degrees of freedom. The asymptotic efficiency of the test against the series of local alternatives $K_n$ is characterized by the non-centrality parameter (21).

The computation of the proposed non-parametric tests based on regression rank scores is illustrated in a simulation study. The behavior of the tests is illustrated in two real-data examples. The power of the test has been estimated under various values of the autoregressive parameter and several different assumptions on innovation distributions, including the symmetric and skew distributions. The results of the numerical studies showed that the proposed test works well for all of these simulation scenarios. In addition, the test works for the Cauchy distribution, even when it does not fulfill our conditions. Therefore, the conjecture says that asymptotic results can be proven under weaker conditions, while the proof is still an open problem.

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## Declarations

**Conflict of interest** The authors have no conflict of interest to declare.

## References

Alpuim T, El-Shaarawi A (2008) On the efficiency of regression analysis with AR(p) errors. J Appl Stat 35(7):717–737

Azzalini A (1985) A class of distributions which includes the normal ones. Scand J Stat 12(2):171–178

Azzalini A (1986) Further results on a class of distributions which includes the normal ones. Statistica (Bologna) 46(2):199–208

Azzalini A (2005) The skew-normal distribution and related multivariate families. Scand J Stat 32(2):159–188

Azzalini A (2023) The R package `sn`: The skew-normal and related distributions such as the skew-*t* and the SUN (version 2.1.1). URL https://cran.r-project.org/package=sn. Home page: http://azzalini.stat.unipd.it/SN/

Basu S, Michailidis G (2015) Regularized estimation in sparse high-dimensional time series models. Ann Stat. https://doi.org/10.1214/15-aos1315

Gutenbrunner C, Jurečková J (1992) Regression rank scores and regression quantiles. Ann Stat 20(1):305–330

Gutenbrunner C, Jurečková J, Koenker R, Portnoy S (1993) Tests of linear hypotheses based on regression rank scores. J Nonparametric Stat 2(4):307–331

Hallin M, Jurecková J (1999) Optimal tests for autoregressive models based on autoregression rank scores. Ann Stat 27(4):1385–1414

Hallin M, Jurečková J, Kalvova J, Picek J, Zahaf T (1997) Nonparametric AR order identification with application to climatic data. Environmetrics 8:651–660

Hallin M, Jurečková J, Picek J, Zahaf T (1999) Nonparametric tests of independence of two autoregressive time series based on autoregression rank scores. J Stat Plan Inference 75(2):319–330

Hallin M, Jurečková J, Koul HL (2007) Serial autoregression and regression rank scores statistics. In: Nair V (ed) Advances in statistical modeling and inference: essays in honor of Kjell A Doksum. World Scientific, Singapore, pp 335–362

Hyndman RJ, Khandakar Y (2008) Automatic time series forecasting: the forecast package for r. J Stat Softw 27:1–22

Jurečková J, Arslan O, Güney Y, Picek J, Schindler M, Tuac Y (2023) Nonparametric tests in linear model with autoregressive errors. Metrika 86(4):443–453

Koenker R (2021) quantreg: quantile regression. https://CRAN.R-project.org/package=quantreg. R package version 5.86

Koenker R, Bassett G Jr (1978) Regression quantiles. Econometrica 46(1):33–50

Koenker RW, d'Orey V (1987) Algorithm as 229: computing regression quantiles. Appl Stat 36:383–393

Koenker R, d'Orey V (1994) A remark on algorithm as 229: computing dual regression quantiles and regression rank scores. Appl Stat Lond 43(2):410–414

Koul HL, Saleh AK Md E (1995) Autoregression quantiles and related rank-scores processes. Ann Stat 23(2):670–689

Kutner MH, Nachtsheim CJ, Neter J (1983) Applied linear regression models. McGraw-Hill/Irwin, New York

R Core Team (2024) R: a language and environment for statistical computing. R Foundation for Statistical Computing, Vienna. https://www.R-project.org/

Ruppert D, Carroll RJ (1980) Trimmed least squares estimation in the linear model. J Am Stat Assoc 75(372):828–838

TCMEB (2021) National education statistics formal education 2020/'21'. https://sgb.meb.gov.tr/www/icerik_goruntule.php?KNO=508. Accessed 03 Dec 2021

Tuaç Y, Güney Y, Şenoğlu B, Arslan O (2018) Robust parameter estimation of regression model with AR(p) error terms. Commun Stat Simul Comput 47(8):2343–2359
Wei X, Liu X, Fan Y, Tan L, Liu Q (2022) A unified test for the AR error structure of an autoregressive model. Axioms 11(12):690
Yee TW (2010) The VGAM package for categorical data analysis. J Stat Softw 32:1–34
Yue S, Pilon P (2003) Interaction between deterministic trend and autoregressive process. Water Resour Res 39(4):96
Yue S, Pilon P, Phinney B, Cavadias G (2002) The influence of autocorrelation on the ability to detect trend in hydrological series. Hydrol Process 16(9):1807–1829