Quantum-like Model of a Rat in a Maze[1]

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Abstract

Quantum mechanics (QM) provides a formal framework for modelling uncertainty and dynamic evolution in physical systems. While its mathematical structure is well established, its application beyond microscopic phenomena remains an area of active discussion. This work illustrates the axioms of QM in an intuitive, accessible way through a textbook-style example designed to parallel decision-making tasks.

The seemingly trivial setup raises questions about the interpretation and understanding of the underlying model. While it does not aim to introduce new results in quantum theory, it serves as a conceptual bridge, demonstrating fundamental principles in a context involving a living organism. The approach may prove useful as a foundation for quantum-inspired models of decision-making and living systems. Links to contemporary interpretations of QM are also discussed.

Keywords: quantum mechanics, quantum-like model, expressing non-physical properties via quantum model, models of living matter.

Notation

Notation is introduced along the text, Table 1 just summarises key objects for later reference. Bra-ket notation [1] is used.

Object	Symbol/font used
Set	$\mathbf{A}, \mathbf{B}, \ldots$
Set of real numbers	$\mathbf{R}$
Set of complex numbers	$\mathbf{C}$
Hilbert space	$\mathcal{H}$
Element from Hilbert space	$\lvert\psi\rangle$
Operator on Hilbert space	$\hat{\mathrm{A}}, \hat{\mathrm{B}}, \ldots$
Matrix representing the operator	$\mathbb{A}, \mathbb{B}, \ldots$

Table 1: Mathematical objects used and their notion.

Introduction: Why rat? Why quantum?

Since we deal with quantum-like models for the tasks of dynamic decision-making (DM) [2], we came up with a simple example to illustrate how the postulates of quantum mechanics (QM) can be applied to a macro-scaled object, and what the technical and (more importantly) interpretational consequences are.

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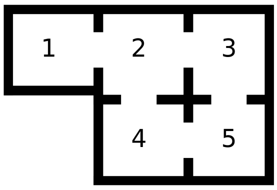

Figure 1: The box rat lives in.

Later became clear that such an example can serve as an environment for simulating more complex aspects of quantum theory.

1 The setup: Rat-in-the-box system

A closed, non-transparent box is divided into several rooms (see Fig.1). A rat is moving between those rooms in discrete steps (the position of the rat is represented by the number of the room). Only some rooms share a common door. The rat can only move through one door at a time (i.e. it takes her several steps to reach distant rooms). The rat is a completely normal rodent; she[2] cannot physically be in several rooms at once.

The rat starts in Room 1 (initial condition) and prefers to stay around Room 5, since it's the room where she is usually fed.

Let us model the rat-in-the-box system using quantum mechanics formalism. Since the box consists of 5 rooms, the state of the rat in the box can be represented[3] as a vector in a 5-dimensional Hilbert space $\mathcal{H}$:

$$\mathcal{H} = \text{span}\left\{|\text{Room}_1\rangle, |\text{Room}_2\rangle, |\text{Room}_3\rangle, |\text{Room}_4\rangle, |\text{Room}_5\rangle\right\} \tag{1}$$

and the general state of the rat can be expressed as a linear combination:

$$|\psi(t)\rangle = c_1(t)|\text{Room}_1\rangle + c_2(t)|\text{Room}_2\rangle + c_3(t)|\text{Room}_3\rangle + c_4(t)|\text{Room}_4\rangle + c_5(t)|\text{Room}_5\rangle, \tag{2}$$

where $c_i(t) \in \mathbf{C}$ are complex coefficients satisfying the normalization condition:

$$\sum_{j=1}^{5} |c_j(t)| = 1 \qquad \text{for each } t \text{ fixed.} \tag{3}$$

Numeric representation of rooms (basis in $\mathcal{H}$ represented by matrix $\mathbb{B}(0) \equiv \mathbb{I}$) could be:

$$\mathcal{H} = \text{span}\left\{ \begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix} \right\}, \tag{4}$$

where the j-th room is represented by $|\text{Room}_j\rangle \equiv |e_j\rangle$. Using (4), (2) reads:

$$|\psi(t)\rangle = c_1(t)\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix} + c_2(t)\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix} + c_3(t)\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix} + c_4(t)\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix} + c_5(t)\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix} = \begin{pmatrix}c_1(t)\\c_2(t)\\c_3(t)\\c_4(t)\\c_5(t)\end{pmatrix}. \tag{5}$$

[2]We refer to the rat in this story as *she/her*.

[3]This paper and this section especially demonstrate a complex approach using very simple algebra. That is the reason why trivial algebraic expressions are rewritten here in (unusually) detailed way.

Since the state is changing in time t, we assume coefficients of superposition are time dependent: $c_j(t)$. This can be expressed alternatively: the basis can be time dependent: $\mathbb{B}(t)$, i.e. due to time evolution the basis "rotates" while coefficients $c_j(t) = c_j$ remains constant.

Observable

Performing a measurement (an observation) means observing the rat's position, i.e., the number of the room the rat is currently in {`'1', '2', '3', '4', '5'`}. Corresponding observable is represented by Hermitian[4] operator $\hat{\mathrm{A}} : \mathcal{H} \to \mathcal{H}$ (in this case represented by matrix $\mathbb{A} \in \mathbf{R}^{5\times 5}$). Spectrum of operator $\hat{\mathrm{A}}$ is $\sigma(\hat{\mathrm{A}}) = \{1, 2, 3, 4, 5\}$, i.e. it is formed from a set of possible outcomes[5].

Probability of observing the outcome

Since the rat's movement is stochastic, we can only think about the probability of finding the rat in the j-th room if the system is in state $|\psi(t)\rangle$. According to Born's rule, the probability is

$$P\Big(\text{rat is in the } j\text{-th room}\Big|\, |\psi(t)\rangle\Big) = P\Big(\alpha_j\Big|\, |\psi(t)\rangle\Big) = \Big\|\hat{\mathrm{E}}(\alpha_j)\,|\psi(t)\rangle\Big\|^2, \tag{6}$$

where α_j is the j-th eigenvalue (representing the j-th outcome of an observation). $\hat{\mathrm{E}}(\alpha_j) : \mathcal{H} \to \mathcal{H}$ is a projector to the j-th eigen-direction $|e_j\rangle$:

$$\hat{\mathrm{E}}(\alpha_j) = |e_j\rangle\,\langle e_j|\,, \tag{7}$$

which suddenly gives a new meaning to the coefficients in (5):

$$P\Big(\text{rat is in the } j\text{-th room}\Big|\, |\psi(t)\rangle\Big) = |c_j(t)|^2. \tag{8}$$

1.1 State update by an observation

The observation process itself influences the state of the rat-in-the-box system. State update can be done according to Lüders formula (also known as Projection postulate of QM [3]):

$$|\psi_{\alpha_j}(t)\rangle = \frac{\hat{\mathrm{E}}(\alpha_j)\,|\psi(t)\rangle}{\Big\|\hat{\mathrm{E}}(\alpha_j)\,|\psi(t)\rangle\Big\|}. \tag{9}$$

Time evolution

In QM setup time evolution is described via Schrödinger equation

$$\begin{cases} i\hbar\dfrac{\mathrm{d}}{\mathrm{d}t}\,|\psi(t)\rangle = \hat{\mathrm{H}}\,|\psi(t)\rangle \\ |\psi(0)\rangle = |\psi_0\rangle \end{cases} \tag{10}$$

[4] Operator $\hat{\mathrm{A}}$ is Hermitian $\Leftrightarrow \hat{\mathrm{A}} = \hat{\mathrm{A}}^*$ (where in the case of matrix representation, star notes transposition and complex conjugation).

[5] This is also the reason why the observable is represented by a matrix of dimension 5: it has to have 5 different eigenvalues.

or alternatively by unitary operator $\hat{\mathrm{U}} : \mathcal{H} \to \mathcal{H}$ in the following way:

$$\begin{cases} |\psi(t)\rangle = \hat{\mathrm{U}}(t)\,|\psi(0)\rangle \\ |\psi(0)\rangle = |\psi_0\rangle\,, \end{cases} \tag{11}$$

where $|\psi_0\rangle \in \mathcal{H}$ is an initial condition and

$$\hat{\mathrm{U}}(t) = \exp\left(-it\frac{1}{\hbar}\hat{\mathrm{H}}\right). \tag{12}$$

In (10)–(12):

- i is a complex unit, $\hbar$ is reduced Planck's constant, which can be omitted since it has no meaning in the considered setup
- t is time ($\hat{\mathrm{U}}$ is time-dependent)
- $\exp(\bullet)$ is exponential of an operator (in this case – exponential of a matrix[6])
- $\hat{\mathrm{H}}$ is Hamiltonian of the system (an operator represented by a matrix).

The matrix representing $\hat{\mathrm{H}}$ is Hermitian and positive semi-definite (its eigenvalues are nonnegative). As a consequence, it is not necessary symmetric: its elements are complex numbers. Due to hermicity, the matrix is symmetric in real parts of its elements, but imaginary parts can differ.

For the scope of our rat-in-the-box setup, the Hamiltonian $\hat{\mathrm{H}}$ can be defined as follows.

- **Diagonal elements** represent the energy associated with a specific state, in our case, the "energy level" is associated with the rat being in a specific room. We can assign the following energies: $E_1 = 3$ for Room 1, $E_2 = 2$ for Room 2, $E_3 = 1$ for Room 3, $E_4 = 1$ for Room 4, $E_5 = 0$ for Room 5. This means that Room 1 is the least preferred one (by the rat) and Room 5 the most preferred one. It can be shown that only differences between energy levels matter (adding a constant to each element on the diagonal would not affect observation probabilities).
- **Off-diagonal elements** represents the coupling strength of the states, in the case of the rat, this tells how likely the rat is to go through the door (transition effort between two rooms). Mainly, this should reflect which rooms do not have a direct connection (for instance, Room 1 and Room 3 in Fig. 1).

Let us assign equal coupling strengths $a \in \mathbf{R}$:

$$\begin{cases} |\psi(t)\rangle = \underbrace{\exp\left(-it\begin{pmatrix} 3 & a & 0 & 0 & 0 \\ a & 2 & a & a & 0 \\ 0 & a & 1 & 0 & a \\ 0 & a & 0 & 1 & a \\ 0 & 0 & a & a & 0 \end{pmatrix}\right)}_{\hat{\mathrm{U}}(t)}|\psi(0)\rangle \\ |\psi(0)\rangle = |\psi_0\rangle\,. \end{cases} \tag{14}$$

Note: multiplying the Hamiltonian with a real number changes only time scale (i.e. $\hat{\mathrm{U}}'(t) = \exp\left(-it\left(k\hat{\mathrm{H}}\right)\right) = \exp\left(-i(kt)\hat{\mathrm{H}}\right) = \hat{\mathrm{U}}(kt)$ for $\forall k \in \mathbf{k}$).

[6]Exponential of a matrix is defined as

$$\exp\left(\mathbb{A}\right) = \sum_{j=0}^{+\infty}\frac{\mathbb{A}^j}{j!}. \tag{13}$$

Since Hamiltonian expresses time dynamics of the rat-in-the-box system, it is expected that by a reasonable choice of elements in $\hat{\mathrm{H}}$ we can express the rat's aim – particularly her preference for the rooms (Room5 is the most preferred one, Room1 is the least preferred by her).

Interpretational aspects of the rat

Until this point, we attempted to apply directly the axiomatics of QM to formulate the rat-in-the-box setup.

Now we would like to go through this process again, this time focusing on the interpretational aspects, i.e. we would like to go beyond borrowing a common understanding of the objects we deal with. Authors are aware of the fact that interpretation is a mathematically unprovable aspect of the theory, which changes *neither* the physical nature of modelled problem, nor the mathematics behind the model.

Nevertheless, this example aims towards understanding how QM describes the setup[7] when both observer and a part of the system are consciousness and possess the free will[8].

By the most used definition, an interpretation of QM is an attempt to explain how mathematical objects of QM correspond to the reality that can be experienced.

Objective facts:

1. State function $|\psi(t)\rangle$ is full description of the system which is rat-in-the-box in our case. This means that anyone knowing time evolution operator $\hat{\mathrm{U}}(t)$ and $|\psi\,(t_{\mathrm{fixed}})\rangle$ is able to compute $|\psi(t)\rangle$ for any $t \in \mathbf{R}$. This result is obtained in fully deterministic way by solving (14).
2. $|\psi(t)\rangle$ is mathematically represented by a finite-dimensional vector whose elements are complex numbers.
3. $|\psi(t)\rangle$ is *mathematically* a superposition (=linear combination) of a given basis in $\mathcal{H}$. Since $|\psi(t)\rangle$ depends on time t, either coefficients of superposition $c_j(t)$ are considered to be dependent on t or the basis itself is changing with time $\mathbb{B}(t) = \mathbb{M}(t)\mathbb{B}(0)$, where $\mathbb{M}(t)$ is unitary matrix expressing time basis evolution and $\mathbb{B}(0)$ is the matrix consisting of vectors forming basis in (4). From a mathematical point of view, both of those considerations are equivalent.

Interpretations considered:

1. $|\psi(t)\rangle$ **is expressing real nature of the system.** In our example, it is understood that rat is some "gas" that moves freely through the closed box and distributes over the rooms. In other words, at a fixed time moment the rat can occupy even several rooms partially at once. At the very moment when an observer opens the box, rat is found in a single particular room.

   Idea in short: rat really *is* in a superposition state occupying several rooms at once, and she instantly "condenses" in a single room when measurement is performed.

   Pros:

   ⊕ consistent with quantum understanding of micro physics (and Copenhagen interpretation of QM)

   ⊕ when accepted, no further interpretational problems arise

   Cons:

   ⊖ seems unrealistic regarding any real-world rat (or any known macro object)

[7] Here, setup means the experimental setup. For example, system+observer.

[8] Although observation process in QM works both for conscious and unconscious observer in the same way, we expect that deeper analysis of the living observer's perspective can help to narrow the gap between DM and QM.

⊖ a free will of the rat (a part of the system) is hardly expressible/imaginable

2. **there's no reality without observation:** the system is not real, only observed objects are. In other words: observed object/property exists only at the moment of the observation and due to it. In our example it means that the box is empty when it stays closed (there is *nothing* inside). Only at the moment when observer opens the box the rooms inside and the rat in one of them are formed and shown to the observer.
   Pros:
   - ⊕ accepts there is no (objective) reality when no observation is performed; implicitly it means reality can be anything when not observed
   - ⊕ consistent with relational interpretation of QM [4]

   Cons:
   - ⊖ seems to be violated when 2 observers perform complementary observations on the same system
   - ⊖ introduces very "crazy" image of the physical world which discards any hope of providing an explanation of how the nature works
3. **state vector is observer's mental state regarding the system:** state function $|\psi(t)\rangle$ describes the state of the system relatively to the observer's mind, i.e. $|\psi(t)\rangle$ does not express real physical properties, however it reflects them in some way. It expresses how observer sees (or can see) the system and possibly his expectations/ideas regarding the system. When observation is performed, it influences observer's information about the system. We claim that:
   - Influence of the observation process onto the system (or its part) is negligible in macro-scaled systems (like mentioned rat-in-the-box).
   - Although there can be a direct causal link between the observation and discontinuous state change, it should be understood as the observer's information gain, not physical changes in the system [2].

   This interpretation is similar to the observation process as known from everyday life: a person does not influence TV by watching it, but he/she enriches his/her knowledge by obtaining some information. We tend to understand the observation process in a similar way: the observer is obtaining some information and his mental state (regarding the system) is updated. In many macroscopic tasks the influence on the system due to observation is negligible (for instance watching TV, listening to lectures, measuring height/weight/distance/velocity/...).
   Pros:
   - ⊕ coincides with common human experience from macro world: gaining information
   - ⊕ consistent with the basic concept of QBism [5]: observation is the experience that the observer gets from his action on the system

   Cons:
   - ⊖ unusual from the perspective of many physicists

Concluding remarks

The rat-in-the-box model presented in this paper serves as a simple example of how the formal postulates of quantum mechanics can be applied to describe a macroscopic, decision-oriented system. The formalism provides a coherent mathematical language for expressing uncertainty, state evolution, and observation in a unified way. While the present text focused on the theoretical formulation and interpretational aspects, preliminary numerical experiments have been carried out and will be presented separately. Future work will explore these computational results and extend the model toward more complex, data-driven and quantum-inspired representations of decision-making processes.

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