Range-Space Modification of Predictive Control for Parallel Robots

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Abstract: - The paper deals with a specific modification of predictive control intended for parallel robots. The modification consists in different definition of requirements for a robot movement. Usually, the movement is determined by known trajectory. The modification investigated in the paper takes into account only range (limits) of required robot movement. In such case, the real resultant robot trajectory is not strictly given, but it follows from control design. Such approach can solve manipulation issues, where the accurate achievement of some trajectory is not important, but the robot has to move through known corridor described by appropriate output range (limits). The design generates optimal control actions, which meet the given output range. The presented modification is documented by several examples of described control process applied to one selected model of planar parallel robot.

Key-Words: - Robotics, Parallel Robots, Predictive Control, Range-Space Control, Manipulation

1 Introduction

In industrial applications especially in robotics, there exist a lot of operations, which provide a different displacement of materials, semi or final products, tools or active elements as sensors and cameras. These operations have a manipulation character, at which the accurate achievement of some predetermined trajectory is not, in most of cases, important, but on the other hand a fulfillment of some permitted output range is required. The range or set of ranges are given by definite limits following from space configuration of individual robot, manipulated object and other possible obstacles occurred in the robot workspace.

Solution of manipulation issues is important particularly at parallel robots [1], where the workspace is more limited and achievement of optimal robot movement is more challenging than in case of conventional open-loop robots.

This paper specifically concerns with one solution of described task based on predictive control [2], which takes into account dynamics of one selected parallel robot. At the beginning, the manipulation problem is formulated and model of robot dynamics is defined. In next sections, the predictive control and its range-space modification [7] is explained. Finally, the paper concludes by several illustrative examples.

2 Problem Formulation

As mentioned, the manipulation belongs to the most frequently occurred operations in robotic applications. From robot control point of view, it can be formulated as a controlled movement of robot gripper (robot end-effector) from start to end point through defined free range Fig.1.

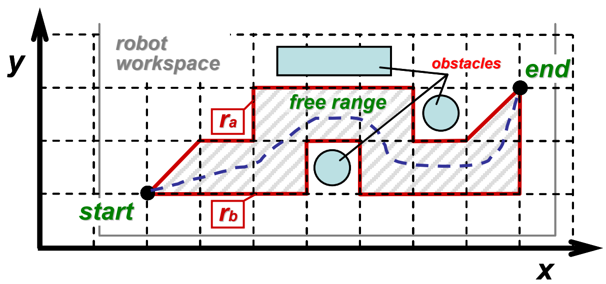

Fig.1 Free range for possible robot movement.

Since, there is no further (exact) specification for the robot movement, the real robot trajectory (trace of realized robot movement) should begin at start point and achieve required end point taking into account ranges of workspace during the robot displacement. It is possible to generate infinity number of such trajectories, but they need not be optimal in view of considered robot. Optimal robot movement can be obtained by specific modification of predictive control design, which includes required points, free (permitted) ranges and model of robot dynamics.

3 Model of Robot Dynamics

Model of robot dynamics expresses time relations of external (drives) and internal (reactions, inertias) forces and moments usually relating to the position of robot gripper (end-effector). By the model, it is possible to predict future robot behavior and to optimize next robot movement.

The robot itself (not only parallel configurations) generally represents multi-body system usually with multi-input multi-output character. It can be straightforwardly described in physical coordinates by Lagrange's equations of mixed type [3]. These equations lead to the system of differential algebraic equations – DAE (1)

$$\begin{aligned} \mathbf{M}\,\ddot{\mathbf{s}} - \mathbf{\Phi}_s^T \boldsymbol{\lambda} &= \mathbf{g} + \mathbf{T}\,\mathbf{u} \\ \mathbf{f}(\mathbf{s}) &= \mathbf{0} \end{aligned} \qquad (1)$$

where $\mathbf{M}$ is a mass matrix, $\mathbf{s}$ is a vector of physical coordinates, $\mathbf{\Phi}_s$ is a Jacobian, $\boldsymbol{\lambda}$ is a vector of Lagrange's multipliers, $\mathbf{g}$ is a vector of other internal relations, matrix $\mathbf{T}$ connects inputs $\mathbf{u}$ to appropriate differential equations and $\mathbf{f}(\mathbf{s}) = \mathbf{0}$ represents geometrical constraints.

As mentioned, the model (1) is a DAE system, since physical coordinates need not represent independent coordinate system. Their number is usually greater than number of degrees of freedom – DOF. However, the model (1) can be transformed to a system of ordinary differential equations in independent coordinates $\mathbf{y}$ corresponding to DOF:

$$\ddot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) + \mathbf{g}(\mathbf{y})\mathbf{u} \qquad (2)$$

where $\mathbf{f}(\mathbf{y}, \dot{\mathbf{y}})$ represents robot dynamics and $\mathbf{g}(\mathbf{y})$ is input matrix of control actions. The model (2) can be rewritten in the state-space formula:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\,\mathbf{u} \\ \mathbf{y} &= \mathbf{H}\,\mathbf{x} \end{aligned} \qquad (3)$$

where $\mathbf{x}$ is a state vector defined as $\mathbf{x} = \begin{bmatrix} \mathbf{y} \\ \dot{\mathbf{y}} \end{bmatrix}$.

This model is simpler and more transparent relating to multi-input multi-output character of robot.

To have finite computation time of the control and to provide uniformly distributed points of robot trajectory, the model (3) is used in discrete form. However, before discretization, it is necessary to provide linearization, since the model contains nonlinear vector function $\mathbf{f}(\mathbf{x})$ in such way, in order to correspond to usual state-space formulation. Matrix $\mathbf{G}(\mathbf{x})$, although is also nonlinear in relation to the robot state $\mathbf{x}$, does not need to be linearized.

The nonlinear function $\mathbf{f}(\mathbf{x})$ in the model (3) can be decomposed according to [9], to have linear form:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}(\mathbf{x})\,\mathbf{x} + \mathbf{G}(\mathbf{x})\,\mathbf{u} \\ \mathbf{y} &= \ \mathbf{H}\ \ \mathbf{x} \end{aligned} \qquad (4)$$

The obtained form (4) represents the robot dynamics identically as model (2) or (3); the individual elements of state and input matrices $\mathbf{A}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ have to be recomputed on-line for appropriate topical robot state $\mathbf{x}$. The used decomposition is described in [9] and its real use is shown in [4] or [7].
Then, the discretization of (4) can be done by usual way via expansion of exponential functions [8]. The resultant model has following standard form:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}(k)\,\mathbf{x}(k) + \mathbf{B}(k)\,\mathbf{u}(k) \\ \mathbf{y}(k) &= \ \mathbf{C}\ \ \mathbf{x}(k) \end{aligned} \qquad (5)$$

where the discrete state is defined as $\mathbf{x}(k) = \begin{bmatrix} \mathbf{y}(k) \\ \dot{\mathbf{y}}(k) \end{bmatrix}$.

This discrete time-variant model represents initial model for control design.

4 Predictive Control

The choice of suitable control depends on character of given control process and used means of control. In robotics, in almost cases, the control process (controlled robot movement) is required to be accurate, robust and effective both in respect time and energy consumption [1].

The robots were and are usually intended for flexible production, therefore their control algorithms have to cope with energy optimization and saving in working time. These algorithms should optimize control actions in relation to future robot behavior.

To managing mentioned conditions, predictive control is offered [2] (Fig.2). It represents a multi-step strategy, which optimizes control actions using quadratic criterion. In it, future robot behavior/states, in defined finite horizon are compared with appropriate requirements. The future states are expressed by equations of predictions based on robot model.

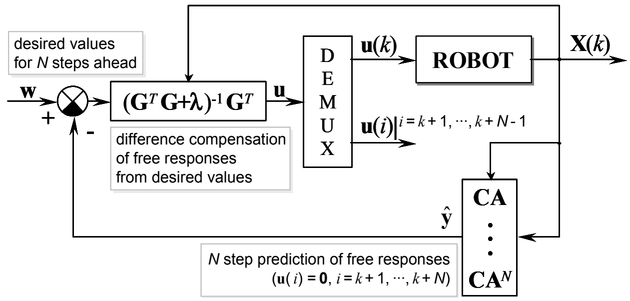

Fig.2 Principal scheme of predictive control.

4.1 Equations of Predictions

Equations of predictions represent from control point of view the expression of feed-forward for given horizon N, which is basis determining dominant part of control actions. Using discrete state-space form (5), the equations have following form [2]:

$$\begin{aligned}
\hat{\mathbf{x}}(k+1) &= \mathbf{A}\,\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\
\hat{\mathbf{y}}(k+1) &= \mathbf{C}\mathbf{A}\,\mathbf{x}(k) + \mathbf{C}\mathbf{B}\mathbf{u}(k) \\
&\vdots \\
\hat{\mathbf{x}}(k+N) &= \mathbf{A}^N\mathbf{x}(k) + \mathbf{A}^{N-1}\mathbf{B}\mathbf{u}(k) + \dots + \mathbf{B}\mathbf{u}(k+N-1) \\
\hat{\mathbf{y}}(k+N) &= \mathbf{C}\mathbf{A}^N\mathbf{x}(k) + \mathbf{C}\mathbf{A}^{N-1}\mathbf{B}\mathbf{u}(k) + \dots + \mathbf{C}\mathbf{B}\mathbf{u}(k+N-1)
\end{aligned} \tag{6}$$

which can be expressed in matrix notation:

$$\hat{\mathbf{y}} = \mathbf{f} + \mathbf{G}\mathbf{u} \tag{7}$$

where $\mathbf{f} = \begin{bmatrix} \mathbf{CA} \\ \vdots \\ \mathbf{CA}^N \end{bmatrix} \mathbf{x}(k)$

and $\mathbf{G} = \begin{bmatrix} \mathbf{C}\mathbf{B} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{C}\mathbf{A}^{N-1}\mathbf{B} & \cdots & \mathbf{CB} \end{bmatrix}$.

The equations are composed by repetitive substitution of unknown future states for states determined from one topical state and state-space model (5). The state-space model is considered to be constant for composition of the equations for considered time step and corresponding horizon N, called horizon of predictions. Control actions, which are occurred in the equations, represent unknown parameters, which are computed by minimization of quadratic criterion.

4.2 Quadratic Criterion

As was mentioned, quadratic criterion is fundamental for determination of control actions. Usually, it is expressed in the following form:

$$J_k = \sum_{j=1}^{N} \left\{ \left\| (\hat{\mathbf{y}}(k+j) - \mathbf{w}(k+j))\mathbf{Q}_y \right\|^2 + \left\| \mathbf{u}(k+j-1)\mathbf{Q}_u \right\|^2 \right\} \tag{8}$$

where N is a horizon of predictions; $\mathbf{Q}_y$ and $\mathbf{Q}_u$ are penalizations; and $\mathbf{w}(k+j)$ are desired values.

The criterion (8) can be expressed also in condensed notation:

$$J_k = [(\hat{\mathbf{y}} - \mathbf{w})^T \ \mathbf{u}^T] \begin{bmatrix} \mathbf{Q_y} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q_u} \end{bmatrix}^T \begin{bmatrix} \mathbf{Q_y} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q_u} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{y}} - \mathbf{w} \\ \mathbf{u} \end{bmatrix} \tag{9}$$

In equation (9), the future outputs $\hat{\mathbf{y}}$ are substituted by use of equation (7). This notation is more advantageous for minimization process.

The control actions are determined on the basis of minimization of described criterion.

4.3 Minimization of the criterion

Used algorithm for minimization of the criterion is crucial for considering of different (additional) control requirements. The simple algorithm based on determination of control actions as a local minimum search ([2], [8]) leads to the following expression

$$\mathbf{u} = \left(\mathbf{G}^T\mathbf{G} + \boldsymbol{\lambda}\right)^{-1}\mathbf{G}^T(\mathbf{w} - \mathbf{f}) \tag{10}$$

However, this algorithm is limited by matrix inversion. Different, more general way is algorithm based on square-root minimization. It arises from condensed notation (9) represented as follows:

$$J_k = \mathbf{J}\,\mathbf{J} \tag{11}$$

from which only one part (square-root) is sufficient to be minimized:

$$\mathbf{J} = \begin{bmatrix} \mathbf{Q_y} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q_u} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{y}} - \mathbf{w} \\ \mathbf{u} \end{bmatrix} \tag{12}$$

After substitution of $\hat{\mathbf{y}}$, the square-root is expressed

$$\mathbf{J} = \begin{bmatrix} \mathbf{Q_y}\,\mathbf{G} \\ \mathbf{Q_u} \end{bmatrix} \mathbf{u} - \begin{bmatrix} \mathbf{Q_y}(\mathbf{w} - \mathbf{f}) \\ \mathbf{0} \end{bmatrix} \tag{13}$$

Its minimization leads to the solution of algebraic equations for unknown control actions

$$\begin{aligned}
\begin{bmatrix} \mathbf{Q_y}\,\mathbf{G} \\ \mathbf{Q_u} \end{bmatrix} \mathbf{u} - \begin{bmatrix} \mathbf{Q_y}(\mathbf{w} - \mathbf{f}) \\ \mathbf{0} \end{bmatrix} &= \mathbf{0} \\
\mathbf{A} \quad \mathbf{u} - \quad \mathbf{b} \quad &= \mathbf{0}
\end{aligned} \tag{14}$$

Obtained vector $\mathbf{u}$ represents control actions for whole horizon N. However, only first appropriate actions are really applied to the robot.

The criterion (8), respectively the form (9) can be simply extended for additional terms representing additional requirements on control. This property will be considered in range-space modification.

The described process of minimization is repeated in every time step for appropriately updated model within defined horizon of prediction.

5 Range-Space Modification

Range-space modification follows from the demand to simplify coding (planning) the robot trajectory in cases, when the trajectory is not determined strictly by some accurate geometrical shape or path. In such cases, there is only information on location of obstacles in a robot workspace and furthermore knowledge of start and required end points [7].

The range-space modification of predictive control can serve either as on-line control for slower movements or as a fast off-line simulative planner of smooth trajectories. To design adequate control actions via range-space modification, it is necessary to make the following steps:

- define free movement ranges
- select suitable penalty matrices

Then, it is possible to minimize modified quadratic criterion.

5.1 Modified Quadratic Criterion

For described design, the predictive control is suitable to be used. It includes robot dynamics and predictions within defined horizon. By this, the optimal distribution of input energy is provided. The quadratic criterion is modified relating to new required ranges [7], which substitute usual desired values

$$J_k = \sum_{j=1}^{N} \left(\left\| \overline{\mathbf{Q}}_{ra} \left(\hat{\mathbf{y}}_{(k+j)} - \mathbf{r}_{a(k+j)} \right) \right\|^2 + \left\| \overline{\mathbf{Q}}_{rb} \left(\hat{\mathbf{y}}_{(k+j)} - \mathbf{r}_{b(k+j)} \right) \right\|^2 + \left\| \overline{\mathbf{Q}}_{\mathbf{u}} \, \mathbf{u}_{(k+j-1)} \right\|^2 \right) \qquad (15)$$

The criterion can be again expressed in advantageous matrix notation

$$J_k = \mathbf{J}\,\mathbf{J} = \|\mathbf{J}\|^2 = \left\| \begin{bmatrix} \mathbf{Q}_{ra} & \cdots & \mathbf{0} \\ \vdots & \mathbf{Q}_{rb} & \vdots \\ \mathbf{0} & \cdots & \mathbf{Q}_{\mathbf{u}} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{y}} - \mathbf{r}_a \\ \hat{\mathbf{y}} - \mathbf{r}_b \\ \mathbf{u} \end{bmatrix} \right\|^2 \qquad (16)$$

The modified criterion leads to the same procedure of its minimization indicated in subsection 4.3, only matrices have different types.

In the criterion, there occur two new terms, which correspond to differences from the limits of free (permissible) ranges. Furthermore, there are also two new output penalizations $\overline{\mathbf{Q}}_{ra}$ and $\overline{\mathbf{Q}}_{rb}$.

In usual control process with standard criterion, the only one appropriate penalization $\mathbf{Q}y$ is selected as identity matrix. However, the new penalizations need not be constant and equal identity matrices. The selection of penalization will be outlined thereinafter in subsection 5.3.

5.2 Definition of free movement ranges

Definition of free movement ranges arises from specific simple envelope of possible free space for robot movement, which excludes all obstacles and other limits of given robot workspace. This envelope is divided into uniform square or triangle segments. The density of segmentation follows from rough distance of the start and end points and selected sampling period. For small periods, the segment density should be higher and vice versa.

Then, the free ranges are determined for individual segments. They represent upper and lower limits for each segment. Important is start and end periods. At the beginning, the range spreads from start point (triangle segment) and at the end it narrows to end point as it is illustrated in Fig.3.

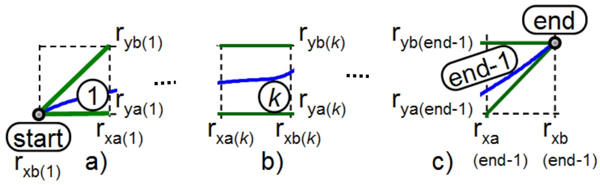

Fig.3 Definition of free ranges
a) acceleration period (start),
b) general state, c) breaking period (end).

Example of one short definition of free ranges is shown in Fig.4. Corresponding values of upper and lower limits of individual segments are enumerated in Table 1. The all values are repeated in number equaled length of horizon N.

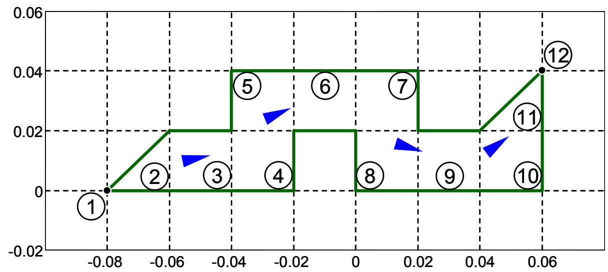

Fig.4 Example of definition of free ranges.

```
%     1      2      3      4      5      6    …
xa=[-0.08  -0.06  -0.04  -0.02  -0.02   0.00  …
xb=[-0.08  -0.08  -0.06  -0.04  -0.04  -0.02  …
ya=[ 0.00   0.02   0.02   0.02   0.04   0.04  …
yb=[ 0.00   0.00   0.00   0.00   0.02   0.02  …

…     7      8      9     10     11     12
…   0.02   0.02   0.04   0.06   0.06   0.06];
…   0.00   0.00   0.02   0.04   0.04   0.06];
…   0.04   0.02   0.02   0.02   0.04   0.04];
…   0.02   0.00   0.00   0.00   0.02   0.04];
```

Table 1 Definition of free ranges:
$\mathbf{r}_a = [\text{xa}, \text{ya}, \psi]$ and $\mathbf{r}_b = [\text{xb}, \text{yb}, \psi]$.

5.3 Selection of Penalty Matrices

Selection of penalty matrices for outputs $\overline{\mathbf{Q}}_{ra}$ and $\overline{\mathbf{Q}}_{rb}$ and for inputs $\overline{\mathbf{Q}}_{u}$ is depended on considered sampling period of the control (speed of control process), selected ranges of segments and horizon of predictions. They have the following structures

$$\overline{\mathbf{Q}}_{ra} = \begin{bmatrix} Qx_{ra} & 0 & 0 \\ 0 & Qy_{ra} & 0 \\ 0 & 0 & Q\psi_{ra} \end{bmatrix}, \quad \overline{\mathbf{Q}}_{rb} = \begin{bmatrix} Qx_{rb} & 0 & 0 \\ 0 & Qy_{rb} & 0 \\ 0 & 0 & Q\psi_{rb} \end{bmatrix},$$

$$\overline{\mathbf{Q}}_{u} = \begin{bmatrix} Qu_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & Qu_m \end{bmatrix} \qquad (17)$$

The structures show details, but it is enough to set only multiplicative constants. They follow from dynamic relations of given robot. Usual adequate selection is $\overline{\mathbf{Q}}_{ra} = \overline{\mathbf{Q}}_{rb} = k_r \, \mathbf{I}$ and $\overline{\mathbf{Q}}_{u} = k_u \, \mathbf{I}$, where $\mathbf{I}$ is identity matrix. The lower values k_r and k_u still keeping control stability lead to the best results.

6 Illustrative Examples

In this section, several simulative examples will demonstrate application of range-space control on one given planar parallel robot 'Moving Slide' considered as basis for simple top milling machine.

6.1 Considered Robot Structure

Considered robot, illustrated in Fig.5, represents horizontal planar parallel mechanism, which includes 4 x Rotational + Prismatic + Rotational joints. Moreover, it is redundantly actuated. There are four drives for only three degrees of freedom. Note, this feature furthermore improves robot stiffness.

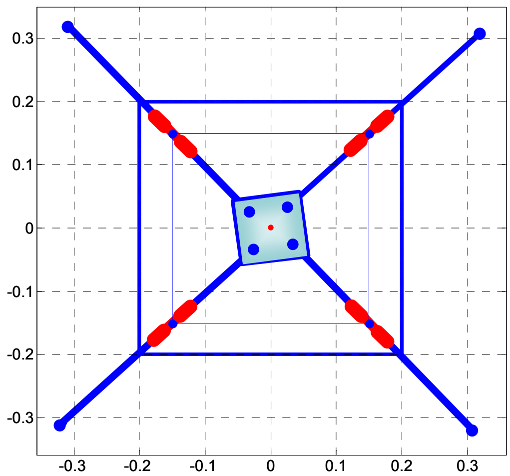

Fig.5 Scheme: 4RPR parallel robot 'Moving Slide'.

6.2 Experiments

The aim of experiments was to control robot through given free ranges and to achieve given end points. The following figures demonstrate such process.

The first, Fig.6, shows realization of free ranges from Fig.4, defined by Table 1.

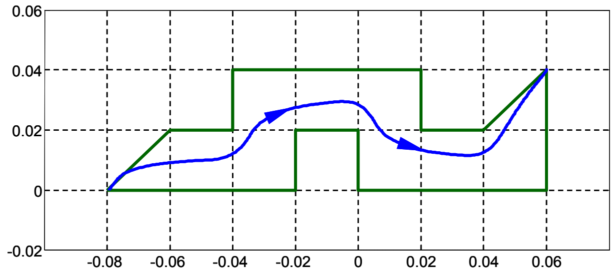

Fig.6 2D plan of realized control process.

The time histories of the corresponding control actions are drawn in Fig.7. In it, the breaking at the end of movement is well perceptible. In some cases, it can be a problem. The robot by its own inertial energy can cross (or miss) the required end point, continues a little bit over and only thereafter stops near the required point. To dampen this undesired property, it is necessary to spread several last segments in higher their number or modify the penalization matrices.

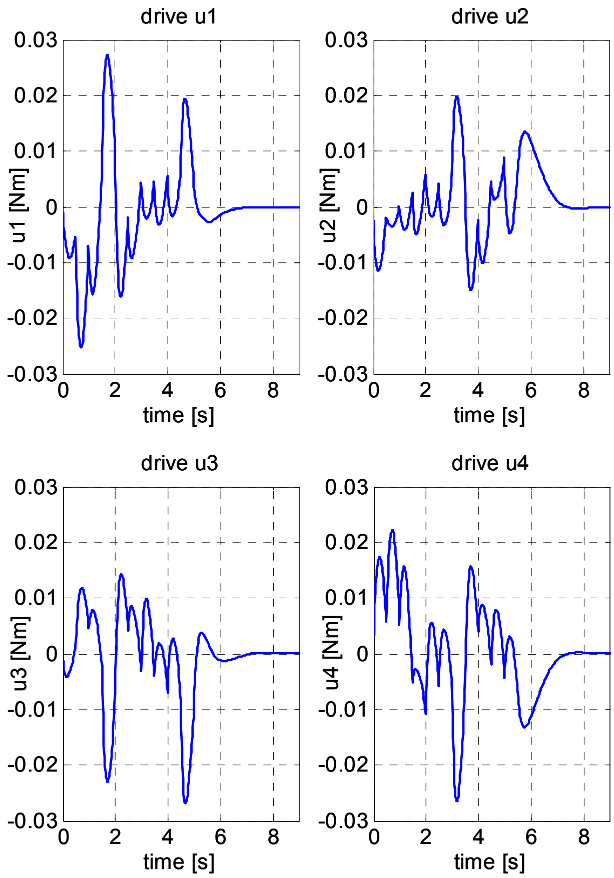

Fig.7 Time histories of control actions for Fig.6.

The further figures (Fig.8 and Fig.9) show other different examples of robot movement.

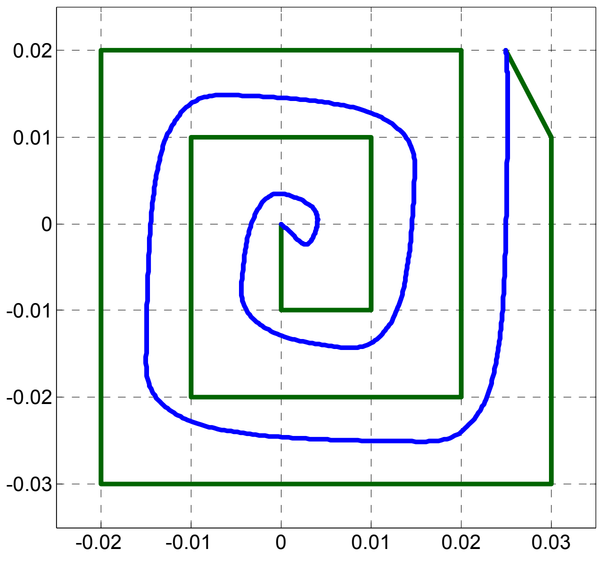

Fig.8 Example of simple spiral trajectory.

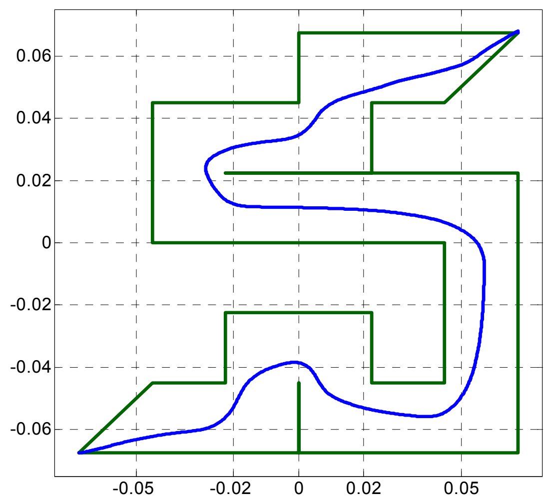

Fig.9 Example of more sophisticated trajectory.

7 Conclusion

In the paper, the range-space modification of predictive control was introduced. It can solve not only mentioned manipulation issues, but also some camera scan of some scene given by ranges (corridors) or can serve after some adjustment for on-line optimal path control of mobile robots.

In general, the predictive control approach belongs to powerful and flexible control strategies, which are promising to solve different requirements of industrial applications.

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