Model – Based Control versus Classical Control for Parallel Robots

Květoslav Belda

Institute of Information Theory and Automation,
Academy of Sciences of the Czech Republic,
Pod vodárenskou věží 4, 182 08 Praha 8 - Libeň.
fax: +420-266052068 and e-mail: belda@utia.cas.cz.

Abstract. Earthly civilization is a complex system based on amount of different relationships. Anybody is not alone. Always, there are some actions that cause some reactions. It happens on different levels. In general, a system can be analyzed from outside as one object with homogeneous properties - global level. The opposite way is analysis from inside - local level. Let us take into account one example of such analysis applied to human work. Specifically, let us focus on comparison of mentioned levels, investigated in control problems of industrial robotics: simple clear local control versus global sophisticated control approach.

This paper outlines the ideas and solutions obtained during design of the control for specific parallel robots. It is demonstrated by comparative simulation of classical PID/PSD controllers - local level and discrete model-based predictive controllers representing global level. Nevertheless, the conclusions of the paper universally show the properties of global and local view on different systems and they are being generalized.

Keywords: PID/PSD feedback controllers, predictive controllers, parallel robotic structures.

1 Introduction

Nowadays, the further development in industrial area is constrained by deficit of powerful machines with adequate dynamics and stiffness (Tsai 1999). Utilization of parallel robots controlled by suitable control algorithms seems to be promising way to improve dynamics, stiffness, accuracy and productivity of machine tools and their centers, which, together with assembly lines, form the backbone of mass production.

The parallel robots – parallel structures (Belda 2002) can be simply understood as movable truss constructions or as movable work platforms supported by several parallel arms. The structures are driven by different drives, in almost cases by electromotors. Simple comparison of serial and parallel structures is shown in Figure 1.

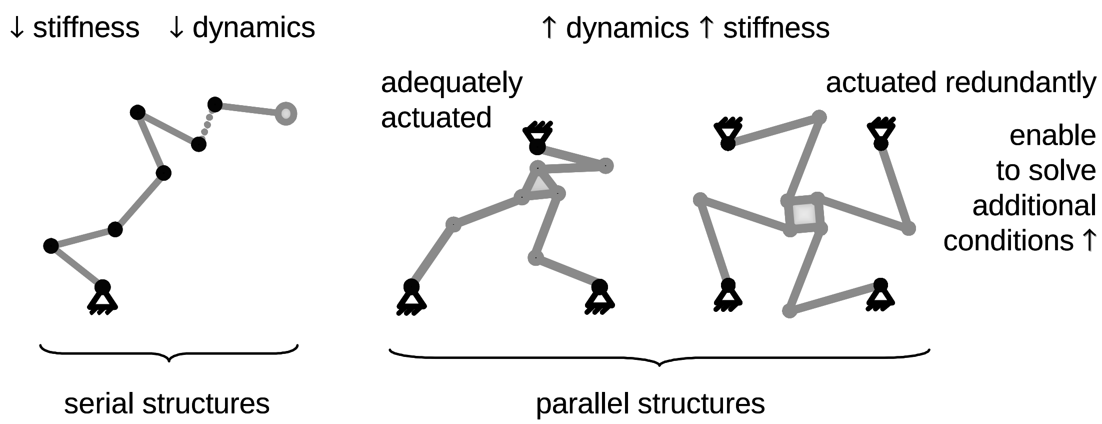

Figure 1. Comparison of serial and parallel structures.

The serial structure has the drives in each joint. The drives in parallel configuration are located in the fixed basic frame.

The fundamental task of the control of mechanical structures driven by electromotors is to design appropriate control actions, which accomplish required movement of the movable platform. In general, the scheme in Figure 2 is solved.

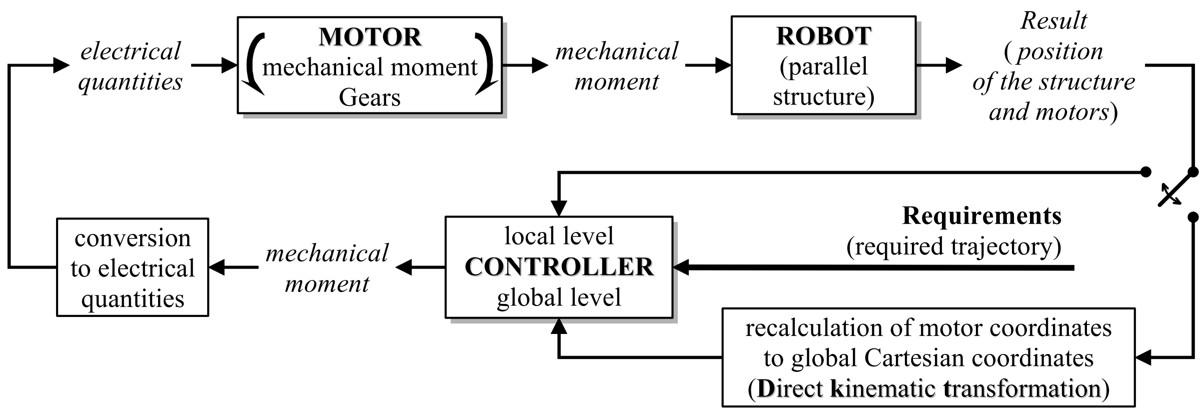

Figure 2. Conceptual scheme of control circuit
for local (independent) and global (centralized) model-based level.

There are two levels of the control (Belda 2002). They are shown in Figure 2: local and global. Local (decentralized) level, represented by continuous PID or discrete PSD controllers, controls each drive independently, without any consideration of mutual relations. The global (centralized) level, represented by predictive controllers (Belda et al. 2002), uses for control the dynamic model of the robot structure. The model can include also drives, which represent also appreciable dynamics. The complete model is used not only for control design, but also for simulation of a real object - robot structure and it can be written in different forms.

In comparison, the classical PID/PSD controllers design the control action only from feedback (Souček 1997). They work up only output information included in feedback regressively without any assumptions to future. On the other hand, the model-based approach combines feedforward and feedback together (Böhm et al. 2001). The model represents prior information that is used for feedforward forming of control actions. Subsequently, the feedback from measurable robot outputs compensates model inaccuracies and certain bounded disturbance. Such design is more optimal and control action can react faster and more effective than PID/PSD structures.

In this paper, firstly, the standard model of the robot is defined (Stejskal et al.). Then, in the main part, the theoretical background of the control design is introduced. It includes new possibilities to utilize the properties of the parallel robots. Finally, the designed control is demonstrated by comparative simulation of the PID/PSD controllers and discrete model-based predictive controllers. Predictive controllers are explained for mechanical and global model including additionally model of drives. In this paper, for simplicity DC motors are used. Thus, let us start with suitable robot model definition.

2 Model for simulation and for control design

2.1 Mechanical model

The robot structure is ideally considered as a system of rigid bodies. In such case, the classical equations of motion can be composed e.g. by utilization of Lagrange's equations. If the obtained model includes dependent coordinates, then they must be removed via knowledge of the null space of the robot jacobian (Stejskal et al. 1996, Belda 2002). Finally, the resultant independent coordinate system (in case of parallel robots, Cartesian system) is the following:

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{x}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{x}} = \mathbf{R}^T\mathbf{g} + \mathbf{R}^T\mathbf{T}\mathbf{u} \quad , \tag{1}$$

where $\mathbf{M}$ is a mass matrix, $\mathbf{g}$ is a vector of right sides, $\mathbf{T}$ is a redistributional matrix, $\mathbf{R}$ is a Jacobian matrix, $\mathbf{x}$ is a robot output (coordinates of a movable work platform, $\mathbf{x} = [x_E, y_E, \psi]$) and $\mathbf{u}$ is an input vector.

The model (1) is a nonlinear system of ordinary differential equations (ODE), and can be rewritten to different forms according to the requirements of appropriate control approach. Since the robot (multibody system) represents relatively time-consuming computation of elements of the model (1), the discrete approach for global model-based strategies is used. It simply respects the time constraints.

The model in the form (1) can be used for simulation of mechanical structure on simple local level. Thus, let us continue with preparation of the model for global level - for predictive controllers. As mentioned previously, we need transform the model to discrete time description. Firstly, to easy describe the structure, we modify the equation system (1):

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{x}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{x}} = \mathbf{R}^T\mathbf{g} + \mathbf{h}\ , \qquad \mathbf{h} = \mathbf{R}^T\mathbf{T}\mathbf{u} \tag{2 a, b}$$

and simplify it to the general form of mechanical systems

$$\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{g}(\mathbf{x})\mathbf{h} \tag{3}$$

Then, the simplification (3), transformed to the state-space formulation, is in a form (4)

$$\begin{aligned}\dot{\mathbf{X}} &= \mathbf{f}(\mathbf{X}) + \mathbf{g}(\mathbf{X})\mathbf{h} \\ \mathbf{x} &= \mathbf{C}\ \mathbf{X}\end{aligned}\ , \tag{4}$$

where the state vector consists of positions and velocities $\mathbf{X} = [\mathbf{x},\ \dot{\mathbf{x}}]^T = [\mathbf{x}_1,\ \mathbf{x}_2]^T$.

(<u>Note</u>: To preserve the traditional control notation in the following text, the symbol **u** is used instead of **h**.)

Now, the formula (4) can be linearized (Valášek et al. 1999) and discretized:

for absolute algorithm of predictive controller:

$$\begin{aligned}\mathbf{X}(k+1) &= \mathbf{A}\ \mathbf{X}(k) + \mathbf{B}\ \mathbf{u}(k) \\ \mathbf{x}(k) &= \mathbf{C}\ \mathbf{X}(k)\end{aligned} \tag{5}$$

or for incremental algorithm of predictive controller ($\mathbf{u}(k) = \mathbf{u}(k-1) + \Delta\mathbf{u}(k)$):

$$\begin{aligned}\mathbf{X}(k+1) &= \mathbf{A}\mathbf{X}(k) + \mathbf{B}\mathbf{u}(k-1) + \mathbf{B}\Delta\mathbf{u}(k) \\ \mathbf{x}(k) &= \mathbf{C}\mathbf{X}(k)\end{aligned} \tag{6}$$

The basis of all predictive controllers is a prediction of new unknown output values **x** from actual topical state **X** in considered horizon of prediction *N*. The prediction of **x** can be defined for both algorithms in common form:

$$\hat{\mathbf{x}} = \mathbf{f} + \mathbf{G}\bar{\mathbf{u}}\ , \tag{7}$$

where **x** is a vector $\hat{\mathbf{x}} = [\hat{\mathbf{x}}(k+1),\ \hat{\mathbf{x}}(k+2),\ \cdots,\ \hat{\mathbf{x}}(k+N)]^T$ (8)

for absolute algorithm $\bar{\mathbf{u}}$ is a vector $\bar{\mathbf{u}} = [\mathbf{u}(k),\ \mathbf{u}(k+1),\ \cdots,\ \mathbf{u}(k+N-1)]^T$ (9)

$$\mathbf{f} = \begin{bmatrix}\mathbf{CA} \\ \vdots \\ \mathbf{CA}^N\end{bmatrix}\mathbf{X}(k),\ \ \mathbf{G} = \begin{bmatrix}\mathbf{C} & \mathbf{B} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{CA}^{N-1}\mathbf{B} & & \cdots & \mathbf{CB}\end{bmatrix} \tag{10}$$

and for incremental algorithm, with $\mathbf{u}(k) = \mathbf{u}(k-1) + \Delta\mathbf{u}(k)$, $\bar{\mathbf{u}}$ is defined as

$$\bar{\mathbf{u}} = \Delta\mathbf{u} = [\Delta\mathbf{u}(k),\ \Delta\mathbf{u}(k+1),\ \cdots,\ \Delta\mathbf{u}(k+N-1)]^T \tag{11}$$

$$\mathbf{f} = \begin{bmatrix}\mathbf{CA} \\ \vdots \\ \mathbf{CA}^N\end{bmatrix}\mathbf{X}(k) + \begin{bmatrix}\mathbf{C\,B} \\ \vdots \\ \mathbf{C}(\mathbf{A}^{N-1} + \cdot + \mathbf{A} + \mathbf{1})\mathbf{B}\end{bmatrix}\mathbf{u}(k-1),\ \ \mathbf{G} = \begin{bmatrix}\mathbf{C} & \mathbf{B} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{C}(\mathbf{A}^{N-1} + \cdot + \mathbf{A} + \mathbf{1})\mathbf{B} & & \cdots & \mathbf{CB}\end{bmatrix} \tag{12}$$

2.2 Global robot model

To obtain the global model of the robot, a model of the drives (electromotors) must be added to the model (1) or to its discrete forms (5) or (6) respectively. As mentioned above, the simple brushes DC motors are considered. Such motors, having permanent magnets in stator, are described by ordinary differential equation of second order (Souček 1997):

$$\ddot{M}_k + \frac{R}{L}\dot{M}_k + \frac{k_{m_1}k_{m_2}}{JL}M_k = \frac{k_{m_1}}{L}\dot{u} \ , \tag{13}$$

where k_{m1} and k_{m2} are torque and speed constants, R and L is terminal resistance and inductance and J is rotor inertia. Equation (13) corresponds with the scheme in Figure 3.

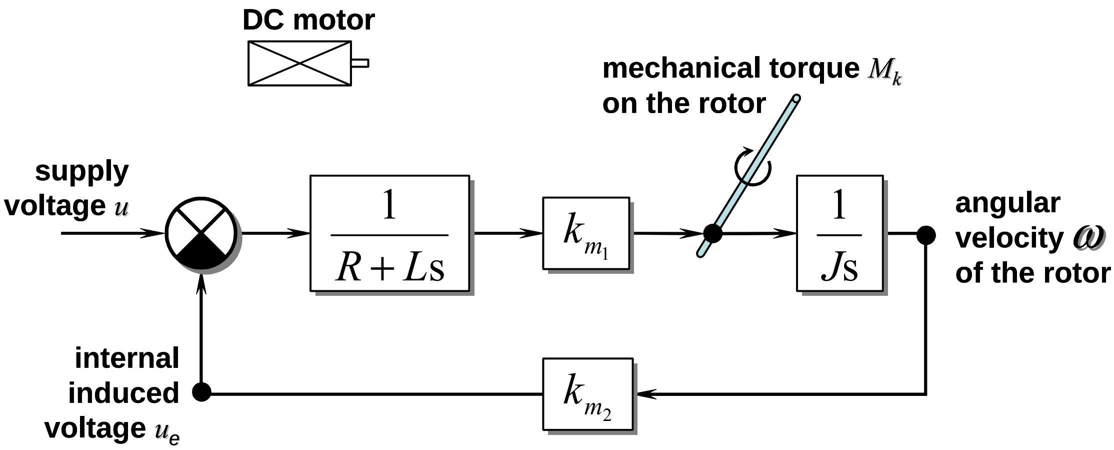

Figure 3. Block scheme of brushes DC motor with permanent magnet in the stator.

In equilibrium case ($\omega = 0$), the order of equation (13) is reduced to first order:

$$L\dot{M}_k + RM_k = k_{m_1}u \tag{14}$$

In transient process, the necessary supply voltage u is a function of load (required) torque $u_r(M_k)$ and angular velocity $u_e(\omega)$ of the rotor.

The global model of whole robot can be composed as follows:

description of mechanical structure:

$$\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{g}(\mathbf{x})\mathbf{M_k} \tag{15}$$

model of the motors:

$$\dot{\mathbf{M}}_\mathbf{k} = -\frac{R}{L}\mathbf{M_k} + \frac{k_{m_1}}{L}\mathbf{u_r} \tag{16}$$

and moreover, the compensation of internal induced voltage u_e:

$$\mathbf{u} = \mathbf{u_r}(\mathbf{M_k}) + \mathbf{u_e}(\boldsymbol{\omega}) = \mathbf{u_r}(\mathbf{M_k}) + k_{m_2}\dot{\boldsymbol{\varphi}} \tag{17}$$

In general, the state-space formulations (5) and (6) changed only in details. The system of equations is only extended by equations of motors and the state vector will include one internal state ($\mathbf{M}_k$) more

$$\mathbf{X} = [\mathbf{x}, \dot{\mathbf{x}}, \mathbf{M}_k]^T = [\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3]^T \tag{18}$$

Finally, the expression for prediction can be written also as equation (7). Now, we can provide fully-valuable control design.

3 Simple local and model-based predictive control

3.1 Local PID/PSD control

The simplest control approach considered means taking the robots and manipulators, powered by group of independent drives /actuators/, separately controlled, as a set of single input - single output systems (setSISO). Mutual interactions among all drives, caused by different positions during the robot movement, are included as disturbances entering each “single” system constituting the robot.

In that view, the classical PID/PSD feedback control scheme can be used. If it is applied, serious problem of mutual conflict of drives may occur (Belda 2002). It is indicated by unpredictable increase of integral/sum (I/S) channels in a controller, caused by the fact, that kinematic description is never perfect. It means, that the description does not represent exactly the real kinematics of redundant parallel structure that is given by production and partly by topical technological conditions.

Moreover, in case of drive redundancy, there exists no unique transformation between coordinates of drives $\boldsymbol{\varphi}$ and independent coordinates $\mathbf{x}$ here. The PID/PSD controllers try to achieve zero errors for all dependent drive coordinates $\boldsymbol{\varphi}$, but it is sometimes impossible. This fact causes the increase of I/S channels in controllers to saturation.

Idea of the solution is the following: local decentralized controllers compute magnitudes of actuators $\mathbf{u}$ for drives and then some operation as a certain projection is applied to these magnitudes (Belda 2002). The projection transforms the actuators to independent space (i.e. it computes so-called general force effects), where the undesirable effects are eliminated, and consecutively the inverse projection recomputes the effects back.

3.2 Model-based predictive control

The derivation of predictive control uses the prediction given by (7). In this paper, the root form is considered. In real computation, it needs matrixes with smaller dimensions and if the penalization $\boldsymbol{\lambda}$ is nonzero value, it keeps redundant properties (if they exist; see Figure 1). Moreover, it can accomplish some additional control requirements (e.g. antibacklash condition, smoothing of torques etc.).

To start derivate Predictive Control in the square - root form, let us define quadratic cost function

$$J_k = \mathcal{E}\left\{ (\hat{\mathbf{x}} - \mathbf{w})^T (\hat{\mathbf{x}} - \mathbf{w}) + \overline{\mathbf{u}}^T \boldsymbol{\lambda}^T \boldsymbol{\lambda}\, \overline{\mathbf{u}} \right\} \tag{19}$$

whose minimization leads to solving the system of algebraic equations:

$$\mathbf{A} \;\; \overline{\mathbf{u}} \;\; - \;\; \mathbf{b} \;\; = \;\; \mathbf{0} \tag{20}$$

For solution, the orthogonal triangular decomposition is used (Lawson et al. 1974). It reduces matrix $\mathbf{A}$ and vector $\mathbf{b}$ to upper triangular matrix $\mathbf{R}$ and vector $\mathbf{c}$ as follows:

$$\mathbf{A}\,\overline{\mathbf{u}} = \mathbf{b} \quad / \mathbf{Q}^T \tag{21}$$

$$\mathbf{A}\,\overline{\mathbf{u}} = \mathbf{b} \;\; \Rightarrow \;\; \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{bmatrix} \overline{\mathbf{u}} = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_z \end{bmatrix} \tag{22}$$

Vector $\mathbf{c}_z$ is a residuum vector, whose Euclidean norm $|\mathbf{c}_z|$ is equal to the square root of cost function (19).

Solving the upper part of the system (22), we obtain either values of supply voltage $\mathbf{u}_r(\mathbf{M}_k)$ or fictitious general force effects $\overline{\mathbf{u}} = \mathbf{h}$ or its increments $\overline{\mathbf{u}} = \Delta\mathbf{h}$, respectively. The values of the voltage can be directly applied (Figure 4). In the second case, when the incremental algorithm is used, the final force effects are given by expression

$$\mathbf{h}(k) = \mathbf{h}(k-1) + \overline{\mathbf{u}}(k)\Big|_{\overline{\mathbf{u}}(k) = \Delta\mathbf{h}(k)} \tag{23}$$

To obtain the real values of actuators **u**, the equation (2 b) must be solved based on fictitious force effects **h** for motor torques **u** (return to initial notation). The solution may not be unique. In general, it represents deficient rank system, where pseudo-inverse operation can be applied (Lawson 1974).

For computation, the values of torques, supply voltage, positions etc. are taken into account in basic units of the system SI (**Mk** [Nm], **u** [V], **x** = [x [m], y [m], ψ [rad]]^T).

(Note: In Figure 4 and also in the following schemes of control circuit, the model of the robot includes also moment of inertia of the rotor of DC motor J [kg·m^{-2}].)

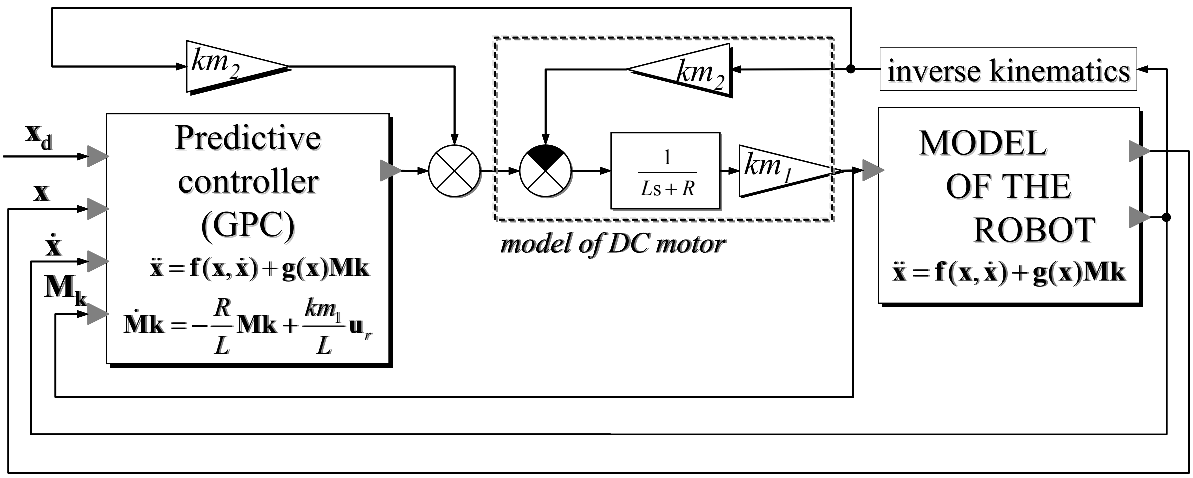

Fig. 4. Scheme of control circuit with predictive controller applied to global model.

Now, let us focus on structure of control shown in Figure 4. There are several feedbacks there. Position feedback (state variable **x**) is the main; current (torque) and velocity feedbacks are inherent and auxiliary feedbacks respectively. Inherent current feedback represents channel of internal state variable **Mk** (**Mk** = f(current $\boldsymbol{i}$ [A])). Auxiliary velocity feedback (state variable $\dot{\mathbf{x}}$) serves the computation of elements of mechanical model and provides compensation of internal induced voltage $\boldsymbol{u_e}$. All mentioned state variables $\mathbf{x}$, $\dot{\mathbf{x}}$ and **Mk** are assumed to be available.

4 Description of the block scheme in SIMULINK environment

The Figure 5 shows parallel simulation of the six different structures of controllers applied to mechanical model of the robot - equations of motion (1).

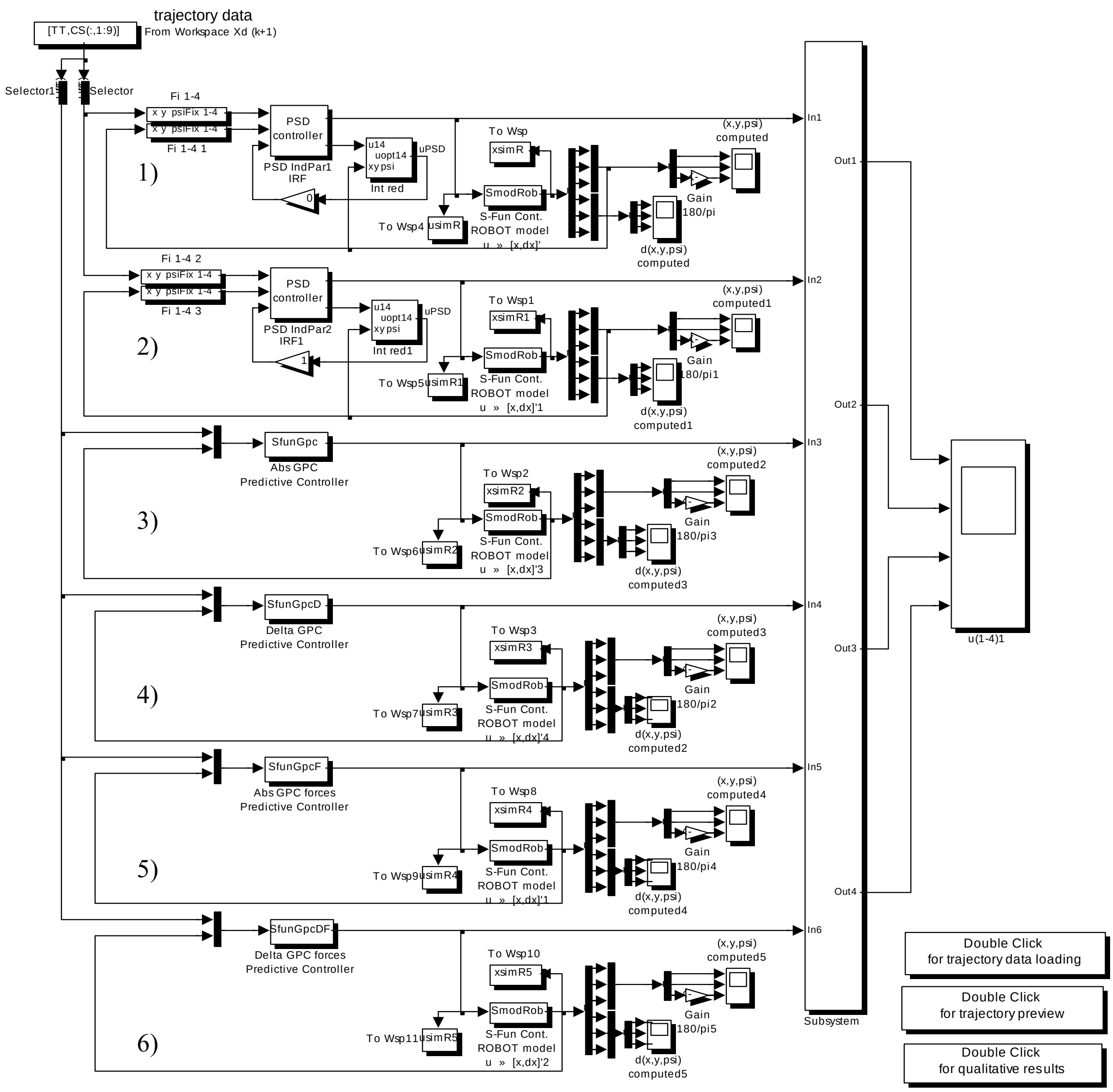

Figure 5. SIMULINK scheme for parallel comparative simulation
of PID/PSD controllers and predictive controllers;

control circuit 1) PID/PSD without compensation of I/S channels;
2) PID/PSD with compensation of I/S channels;
3) and 4) predictive controllers generating the torques directly;
5) and 6) predictive controllers generating control torques via generalized force effects
(3) and 5) absolute algorithms; 4) and 6) incremental algorithms).

The following Figure 6 demonstrates predictive controller, which considers the global robot model (15)-(17), i.e. it takes into account the model of the drives (DC motors).

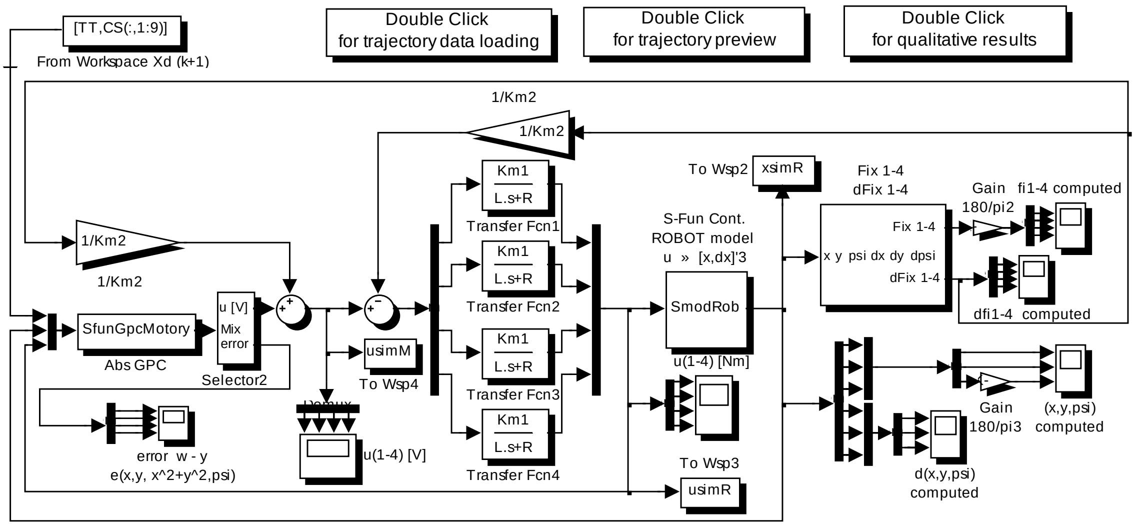

Figure 6. Control circuit with global robot model.

The simple implementation in SIMULINK environment is provided by classically structured M S-functions (predictive controllers - `SfunGpc*.m`, model of the robot - `SmodRob.m` .

5 Comparative example from simulation

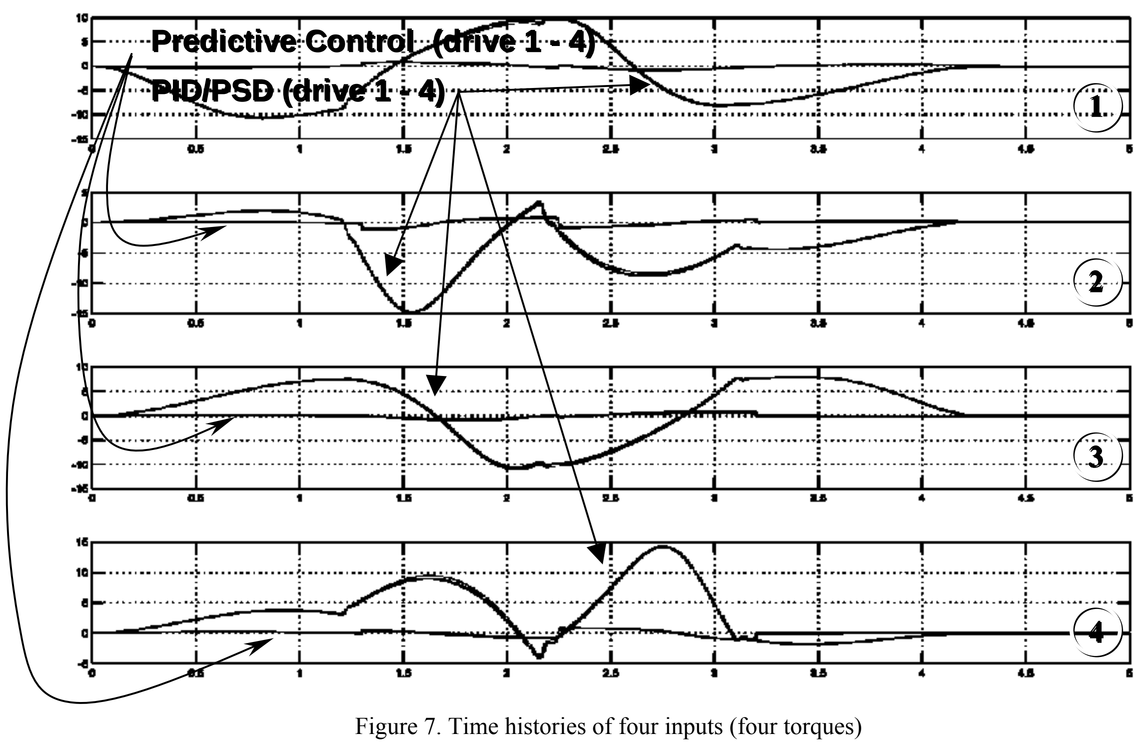

Figure 7. Time histories of four inputs (four torques)
during parallel simulation of the block scheme shown in Figure 4.

The time histories (Figure 7) were recorded during the motion along “S-shaped” trajectory. Figure 7 illustrates all predictive control approaches, with both mechanical and global model. The results of predictive controllers look similar, but in real implementation, are different.

From practical point of view, the setting of parameters of PID/PSD is not simple task. The Figure 7 illustrates the properties of PID/PSD control deducing the actions only from the feedback, which cannot be energetically effective. At the same time, the Figure 7 shows the advantages of the combination of feedforward action modulation with feedback inaccuracy and disturbance compensation – more effective control way with lower energetic demands.

6 Conclusion

The paper deals with a set of possible ways of control and their comparison. It shows that the classical approaches are energetically consuming. They should be, in a future, replaced by such strategies e.g. strategies based on predictive controllers, which consider the most of the available information on given controlled object, in this paper, robotic structure driven by DC motors.

From general point of view the paper shows that the different levels of analysis, both local and global, are important. However, it is necessary to determine the adequate ratio of both levels. In view of paper subject, if we control individual motor used as only one individual drive of the system, the local analysis is meaningful. In case of more number of motors as several drives of one system or system centers, it is necessary to consider both levels – example of parallel robotic structures. Global level provides the optimal distribution of input energy. Then, all drives can cooperate effectively without mutual fighting. Finally, local level performs just their own movement – change of position of individual drive. This change is transformed through the mechanical elements – kinematic structure – to resultant movement – global result.

The similar relation can find in human civilization. Good example is a situation of individual countries in gradually integrating Europe, which, as one complex partnership, tries to compete with United States and countries of eastern Asia.

Acknowledgement

This research is supported by GA ČR (101/03/0620, 2003-2005) "Redundant drives and measurement for hybrid machine tools", GA ČR (102/02/0204, 2002-2004) "Design of adaptive intelligent controllers" and GA ČR (102/01/0608, 2001-2003) "New directions of linear control".

References

Belda, K. (2002), *Control of Redundant Parallel Structures of Robotic Systems*, dissertations, CTU in Prague.

Belda, K., Böhm, J. Valášek, M. (2002) "State-Space Generalized Predictive Control for Redundant Parallel Robots", *NATO ASI Workshop, Praha 2002*, pp. 9-14.

Böhm, J., Belda, K., Valášek, M. (2001) "Study of Control of Planar Redundant Parallel Robot." *Proceedings of the IASTED Int. conference MIC 2001*, *Innsbruck 2001*, pp. 694-699.

Lawson, Ch. J., Hanson, R. J. (1974), *Solving Least Square Problems*, Prentice - Hall, Inc.

Online Manuals (in PDF): External Interfaces; Writing S-Functions. The MathWorks, Inc. 99-01.

Souček, P. (1997), *Drives of Production Appliances*, script of CTU in Prague, (in Czech).

Stejskal, V., Valášek, M. (1996), *Kinematics and Dynamics of Machinery*, Marcel Dekker, Inc.

Tsay, Lung-Wen (1999), *Robot analysis, The Mechanics of Serial and Parallel Manipulators*, John Wiley & sons, Inc.

Valášek, M., Steinbauer, P. (1999) "Nonlinear Control of Multibody Systems", *In: Euromech 99, Lisbon,* pp. 437 - 444.