

# GENERALIZED PREDICTIVE CONTROL FOR REAL-TIME USE IN AREA OF PARALLEL ROBOTS

**Květoslav Belda**

*Institute of Information Theory and Automation,  
Academy of Sciences of the Czech Republic,  
Department of Adaptive Systems.  
Pod vodárenskou věží 4, 182 08 Praha 8 – Libeň, Czech Republic.*

**Abstract:** The paper deals with Generalized Predictive Control for path control of the redundant parallel robots. It summarizes classical and square root minimization of the quadratic criterion and direct and two-step design of actuators respectively. As an example, the planar redundant parallel robot is used. Moreover, the paper presents several possibilities to use predictive algorithm for fulfillment of some additional requirements like actuator smoothing or antibacklash control.

**Keywords:** Predictive control, Redundancy, Quadratic criterion, State formulation.

## 1. INTRODUCTION

The next development in industrial area is constrained by deficit of powerful machines with proportional dynamics and stiffness. At the same time, the new control techniques, which would be able to achieve higher accuracy with keeping of dexterity of the robot, are missing or, on the other hand, there is no interest for their real application in new machines.

Promising way for mentioned problems can be utilization of new robot types based on parallel constructions (Neugebauer (ed.), 2002). Although these constructions raises new questions, especially in control area, their parallel character gives the possibility to significantly improve mechanical parameters of presently being-developed industrial machines (i.e. dynamics, stiffness, kinematic accuracy etc.).

The aim of this paper is an investigation of one promising control approach - Generalized Predictive Control (GPC) (Ordys and Clarce, 1993; Böhm, et al., 2001a) as an example of high-level model-based control. This approach offers to achieve higher accuracy and at the same time effective cooperation of all actuators. Furthermore, it offers several possibilities to realize some additional requirements (Böhm, et al., 2001b), e.g.: smoothing of the actuator trends or fulfillment of antibacklash condition.

The paper initially focuses on model description of the parallel structure, then continues with introduction of predictive control technique and finally shows simulation examples and discusses real-time utilization.

## 2. DESCRIPTION OF THE ROBOT MODEL

The robot (manipulator) is a multibody system, which can be described by Lagrange's equations, in redundant case, of mixed type. These equations lead to the DAE system (the Differential - Algebraic Equations – combination of differential equations with algebraic constraints) in the following form:

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{s}} - \Phi_s^T \boldsymbol{\lambda} &= \mathbf{g} + \mathbf{T}\mathbf{u} \\ \mathbf{f}(\mathbf{s}(t)) &= \mathbf{0} \end{aligned} \quad (1)$$

where  $\mathbf{M}$  is a mass matrix,  $\mathbf{s}$  is a vector of physical coordinates (their number is higher than number of degrees of freedom /DOF/),  $\Phi_s$  is an overall Jacobian of the system,  $\boldsymbol{\lambda}$  are Lagrange's multipliers,  $\mathbf{g}$  is a vector of right sides, matrix  $\mathbf{T}$  transforms the inputs  $\mathbf{u}$  (n torques) into n drives and  $\mathbf{f}(\mathbf{s}(t)) = \mathbf{0}$  represents geometrical constraints.

The physical coordinates  $\mathbf{s}$  consist of the independent coordinates  $\mathbf{x}$  (Cartesian coordinates of the fix point of the cutting tool or gripper), drives' - actuators' coordinates  $\mathbf{q}_1$  and other auxiliary geometrical coordinates  $\mathbf{q}_2$ .

Let us consider the possibility to transform the model (1) into independent coordinates  $\mathbf{x}$  (Stejskal and Valášek, 1996). As follows, the DAE robot model is transformed to the ordinary differential model (ODE). It means that the Lagrange's multipliers disappear and design of the robot control becomes considerably simpler. Then the final model of the robot system is the following:

$$\mathbf{R}^T \mathbf{M} \mathbf{R} \ddot{\mathbf{x}} + \mathbf{R}^T \mathbf{M} \dot{\mathbf{R}} \dot{\mathbf{x}} = \mathbf{R}^T \mathbf{g} + \mathbf{R}^T \mathbf{T} \mathbf{u} \quad (2)$$

It is very important to note, firstly, that the Jacobian matrix  $\mathbf{R}$  is the basis of the null space of the overall Jacobian  $\Phi_s$  and thus it satisfies the expression

$$\Phi_s \mathbf{R} = \mathbf{R}^T \Phi_s^T = \mathbf{0} \quad \text{and} \quad \dot{\mathbf{s}} = \mathbf{R} \dot{\mathbf{x}} \rightarrow \ddot{\mathbf{s}} = \mathbf{R} \ddot{\mathbf{x}} + \dot{\mathbf{R}} \dot{\mathbf{x}} \quad (3)$$

and, secondly, the Jacobian  $\mathbf{R}$  can be decomposed into submatrixes  $\mathbf{R}_{q_1}$ ,  $\mathbf{R}_{q_2}$  and  $\mathbf{R}_x = \mathbf{I}_x$ . Submatrix  $\mathbf{R}_{q_1}$  ( $= (\mathbf{R}^T \mathbf{T})^T$ ) defines important relation between  $\dot{\mathbf{q}}_1$  and  $\dot{\mathbf{x}}$  as

$$\dot{\mathbf{q}}_1 = \mathbf{R}_{q_1} \dot{\mathbf{x}} \quad \left( \equiv \frac{d\mathbf{q}_1}{dt} = \mathbf{R}_{q_1} \cdot \frac{d\mathbf{x}}{dt} \right) \quad (4)$$

which will be useful in section dealing with design of control law in root form.  $\mathbf{R}_1$  can be also obtained from geometrical relation  $\mathbf{q}_1(\mathbf{x})$ :

$$\mathbf{R}_1 = \left[ \frac{\partial \mathbf{q}_1(\mathbf{x})}{\partial x_1}, \dots, \frac{\partial \mathbf{q}_1(\mathbf{x})}{\partial x_n} \right] \Bigg|_{\substack{n = \text{number of independent} \\ \text{coordinates} = \\ = \text{degrees of freedom}}} \quad (5)$$

### 3. CLASSICAL DESIGN OF CONTROL LAW

The principal task of control of the robots is accomplishment of their movement along a planned trajectory (technological requirements). In some cases, it is very sophisticated and difficult for general control approaches like classical PID structures. Therefore, the new control approaches, which are being developed, are directly adjusted for concrete system (machine, robot). High-level controls, which use knowledge of the mathematical model e.g. equations (1, 2), represent suitable approach, which takes into account dynamic trend of the controlled system. In this way, it can better comply with mentioned requirements from technology. On the base of the dynamic model, equation (2), high-level controls globally optimize whole control process and can predict future actions. One of them is Generalized Predictive Control (GPC).

The Predictive control (Ordys and Clarce, 1993; Böhm, et al., 2001a) is a multi-step control based on local optimisation of the quadratic criterion, where the linearized equation or state formula is used (i.e. only the nearest future control signal is evaluated). This approach admits combination of feedback~feedforward parts.

For design of predictive control law, the nonlinear model (2) must be linearized (Valášek and Steinbauer, 1999) and converted from continuous to discrete time. This described model transformation enables us to consider the classical discrete state formula in the following form:

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A} \mathbf{X}(k) + \mathbf{B} \mathbf{u}(k) \\ \mathbf{x}(k) &= \mathbf{C} \mathbf{X}(k) \end{aligned} \quad (6)$$

where  $\mathbf{X}$  is composed as  $\mathbf{X} = [\mathbf{x}, \dot{\mathbf{x}}]^T$  and  $\mathbf{x}$  agrees with equation (2). Furthermore for law derivation, the expression of new unknown output values  $\mathbf{x}$  from topical state  $\mathbf{X}$  is needed. The following lines imply this expression.

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{C} \mathbf{X}(k) \\ \hat{\mathbf{X}}(k+1) &= \mathbf{A} \mathbf{X}(k) + \mathbf{B} \mathbf{u}(k) \\ \hat{\mathbf{x}}(k+1) &= \mathbf{C} \mathbf{A} \mathbf{X}(k) + \mathbf{C} \mathbf{B} \mathbf{u}(k) \\ &\vdots \\ \hat{\mathbf{X}}(k+N) &= \mathbf{A}^N \mathbf{X}(k) + \mathbf{A}^{N-1} \mathbf{B} \mathbf{u}(k) + \dots + \mathbf{B} \mathbf{u}(k+N-1) \\ \hat{\mathbf{x}}(k+N) &= \mathbf{C} \mathbf{A}^N \mathbf{X}(k) + \mathbf{C} \mathbf{A}^{N-1} \mathbf{B} \mathbf{u}(k) + \dots + \mathbf{C} \mathbf{B} \mathbf{u}(k+N-1) \end{aligned}$$

then the prediction of  $\mathbf{x}$  is the following

$$\hat{\mathbf{x}} = \mathbf{f} + \mathbf{G} \mathbf{u} \quad (7)$$

$$\mathbf{f} = \begin{bmatrix} \mathbf{C} \mathbf{A} \\ \vdots \\ \mathbf{C} \mathbf{A}^N \end{bmatrix} \mathbf{X}(k) \quad \text{and} \quad \mathbf{G} = \begin{bmatrix} \mathbf{C} & \mathbf{B} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C} \mathbf{A}^{N-1} \mathbf{B} & \dots & \mathbf{C} \mathbf{B} \end{bmatrix} \quad (8)$$

Now we can optimize the quadratic criterion. The criterion is optimized in instant  $k$ , with using predictions of  $\mathbf{x}$  ( $\hat{\mathbf{x}} = [\hat{\mathbf{x}}_{k+1} \cdots \hat{\mathbf{x}}_{k+N}]^T$ )

$$\begin{aligned} J_k &= \mathcal{E}\{(\hat{\mathbf{x}} - \mathbf{w})^T (\hat{\mathbf{x}} - \mathbf{w}) + \mathbf{u}^T \boldsymbol{\lambda} \mathbf{u}\} = \\ &= \mathcal{E}\{(\mathbf{G} \mathbf{u} + \mathbf{f} - \mathbf{w})^T (\mathbf{G} \mathbf{u} + \mathbf{f} - \mathbf{w}) + \mathbf{u}^T \boldsymbol{\lambda} \mathbf{u}\} \end{aligned} \quad (9)$$

where  $\mathcal{E}$  is an operator of mean value,  $N$  is a horizon of prediction,  $\mathbf{x}$  is a vector of outputs,  $\mathbf{w}$  are desired values,  $\boldsymbol{\lambda}$  is a penalization of input and  $\mathbf{u}$  is a vector of robot inputs. Considering the condition of optimization

$$J_k \stackrel{!}{=} \min \quad (10)$$

for criterion (9), the resultant control law is

$$\mathbf{u} = (\mathbf{G}^T \mathbf{G} + \boldsymbol{\lambda})^{-1} \mathbf{G}^T (\mathbf{w} - \mathbf{f}) \quad (11)$$

This control law (11) can be already used. It should be noted that only the first element  $\mathbf{u}(k)$  from vector  $\mathbf{u}$  is used. If penalization  $\boldsymbol{\lambda}$  is greater than zero, then the matrix  $\mathbf{G}^T \mathbf{G}$  is regular for all cases, adequately actuated even for redundant cases. Theoretical case of zero penalization  $\boldsymbol{\lambda}$  with redundant robot can be solved by pseudoinversion (Lawson and Hanson, 1974).

#### 4. DESIGN OF CONTROL LAW IN SQUARE ROOT FORM

This chapter aims on derivation of control law for different configuration of elements in mathematical model (2), which needs matrices with smaller dimensions. Moreover, if the penalization is positive, the computation also holds the redundant properties (if exist). It can be also used for accomplishment of additional control requirements.

Furthermore, in this chapter, the advantages of the square root optimization of quadratic criterion (9) are used, marked out by compact notation and good preparation for operations with huge matrices.

Let us proceed from nonlinear differential model (2) and from its simplified form:

$$\begin{aligned} \mathbf{R}^T \mathbf{M} \mathbf{R} \ddot{\mathbf{y}} + \mathbf{R}^T \mathbf{M} \dot{\mathbf{R}} \dot{\mathbf{y}} &= \mathbf{R}^T \mathbf{g} + \mathbf{R}^T \mathbf{T} \mathbf{u} \\ \mathbf{R}^T \mathbf{M} \mathbf{R} \ddot{\mathbf{y}} + \mathbf{R}^T \mathbf{M} \dot{\mathbf{R}} \dot{\mathbf{y}} &= \mathbf{R}^T \mathbf{g} + \mathbf{F} \mathbf{M} \end{aligned} \quad (12)$$

where new vector  $\mathbf{F} \mathbf{M}$  represents new fictitious input to the system so called general forces.

In equation (12) we can apply the same procedures of linearization (Valášek and Steinbauer, 1999), discretization and use the same composition of prediction formula (chapter 3,  $\hat{\mathbf{x}} = \mathbf{f} + \mathbf{G} \mathbf{u}$ , (7)) for future output values.

The quadratic criterion (9) ( $J_k = \mathcal{E} \{ (\hat{\mathbf{x}} - \mathbf{w})^T (\hat{\mathbf{x}} - \mathbf{w}) + \mathbf{u}^T \boldsymbol{\lambda}^T \boldsymbol{\lambda} \mathbf{u} \}$ ) can be rewritten in the root form as a product of matrices

$$J_k = \begin{bmatrix} [\hat{\mathbf{x}} - \mathbf{w}]^T, \mathbf{u}^T \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\lambda} \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\lambda} \end{bmatrix} \begin{bmatrix} [\hat{\mathbf{x}} - \mathbf{w}] \\ \mathbf{u} \end{bmatrix} = \mathbf{J}^T \mathbf{J} \quad (13)$$

Now we can work only with root of the criterion

$$\mathbf{J} = \begin{bmatrix} \hat{\mathbf{x}} \\ \boldsymbol{\lambda} \mathbf{u} \end{bmatrix} - \begin{bmatrix} \mathbf{w} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{G}\mathbf{u} + \mathbf{f} \\ \boldsymbol{\lambda} \mathbf{u} \end{bmatrix} - \begin{bmatrix} \mathbf{w} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{G} \\ \boldsymbol{\lambda} \end{bmatrix} \mathbf{u} + \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{w} \\ \mathbf{0} \end{bmatrix} [= \mathbf{0}] \quad (14)$$

$$\mathbf{A} \mathbf{u} - \mathbf{b} = \mathbf{0} \quad (15)$$

and consecutively we look for such action  $\mathbf{u}$ , which minimizes root form (14, 15), i.e. we look for  $\mathbf{u}$ , in order to minimize the norm  $|\mathbf{J}|$ . If we annul the root of criterion (14), we will obtain system of equation (15) with more rows than columns (over-determined system).

For computation, the triangular-orthogonal decomposition [6] is used. It reduces excess rows of the matrix  $\mathbf{A}$   $[(2 \cdot N \cdot n) \times (N \cdot n)]$  and elements of vector  $\mathbf{b}$   $[2 \cdot N \cdot n]$  ( $n$  is a number of DOF) into upper triangle  $\mathbf{R}$  and shorter vector  $\mathbf{c}_1$  according to the following scheme:

$$\begin{bmatrix} \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{b} \end{bmatrix} \Leftrightarrow \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_z \end{bmatrix} \quad (16)$$

Vector  $\mathbf{c}_z$  is a loss vector. Its Euclidean norm  $|\mathbf{c}_z|$  is equal to root of quadratic criterion; scalar  $\sqrt{J}$  (i.e.  $J = \mathbf{c}_z^T \mathbf{c}_z$ ).

For solution, we need only the upper part of the system of equations (16), which can be simply solved in view of the vector of actuators  $\mathbf{u}$  by backward-run procedure.

Obtained actuators represent fictitious generalized force effects  $\mathbf{u}$ , from which only the first subvector (for  $k$  instant)  $\mathbf{u}(k) = \mathbf{FM}$  is used. It must be recomputed, according to substitution in equations (12), to really used actions (drives):

$$\mathbf{R}^T \mathbf{T} \mathbf{u}_{(\text{drives})} = \mathbf{FM} \quad (17)$$

with the same meaning of matrices  $\mathbf{R}$  and  $\mathbf{T}$  as in the system of differential equations (12). System (17) generally expresses deficient rank equation system (lower number of rows than columns i.e. than unknown real inputs - actions). There is again possibility to use pseudo-inverse of the matrix  $\mathbf{R}^T \mathbf{T}$  there.

## 5. EXAMPLES AND CONCLUSIONS

This section shows different actuators' trends for different control requirements, applied to planar redundant parallel robot (Fig. 1), for one selected trajectory.

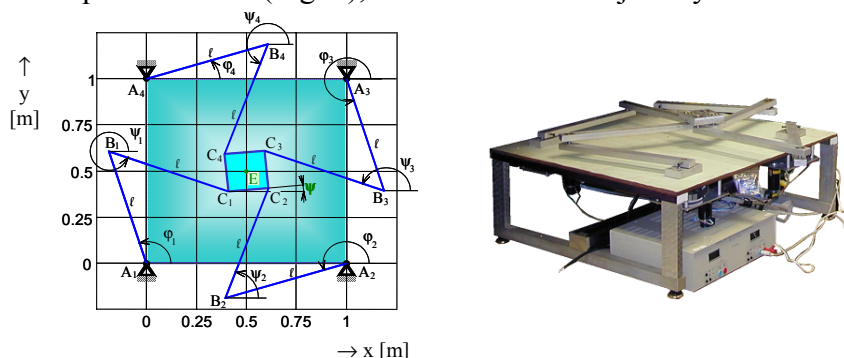


Fig. 1. Scheme and laboratory model of the parallel robot.

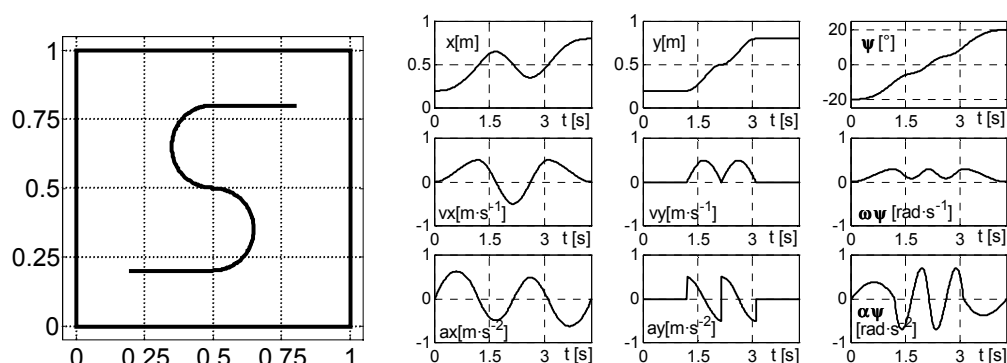


Fig. 2. Example of one planned trajectory with its kinematic characterizations.

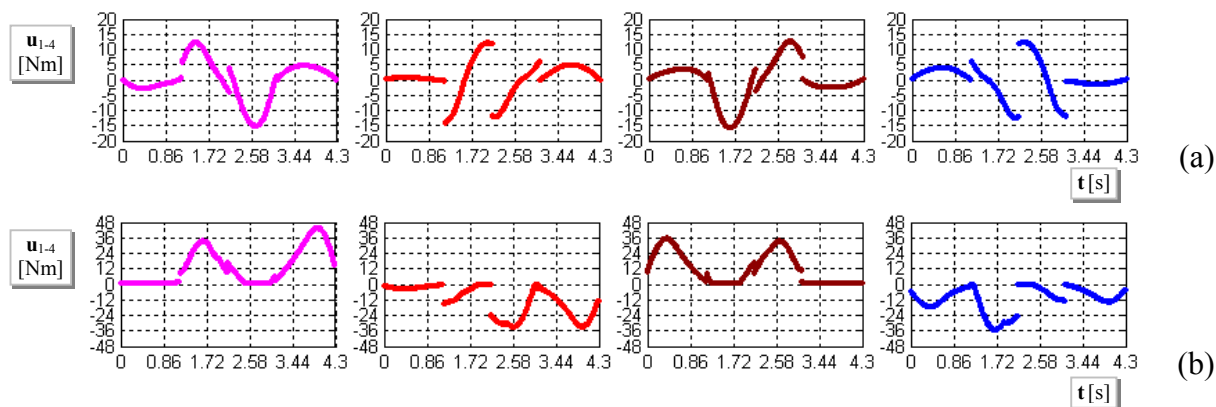


Fig. 3. Trends of four actuators unlimited (a) and limited (b) by antibacklash condition (sampling  $T_s = 0.01s$ ; max. error  $1 \mu m$ ; penalization  $\lambda = 10^{-12}$ ; horizon  $N = 10$ ).

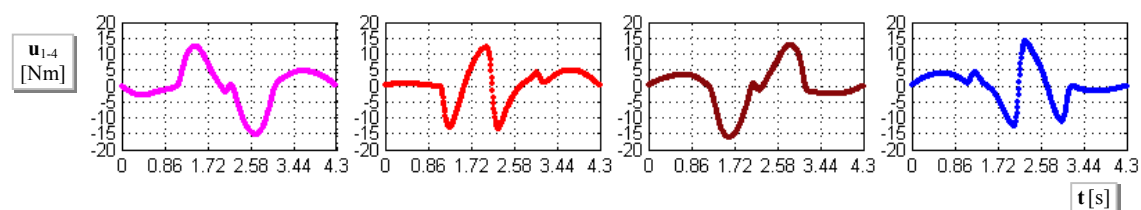


Fig. 4. Smoothing of the actuators trends for trajectory from Fig. 2. (sampling  $T_s=0.01s$ ; max. error  $2.02 \text{ mm}$ ; penalization  $\lambda = 5 \cdot 10^{-8}$ ; horizon  $N = 10$ ).

The second, square root approach is more suitable for real application (Fig. 5 and Fig. 6), because it represents less mathematical operations than classical approach. At present, it is tested on real laboratory model with the same structure as in Fig. 1. From result viewpoint, the both approaches, classical and square root algorithms, are identical.

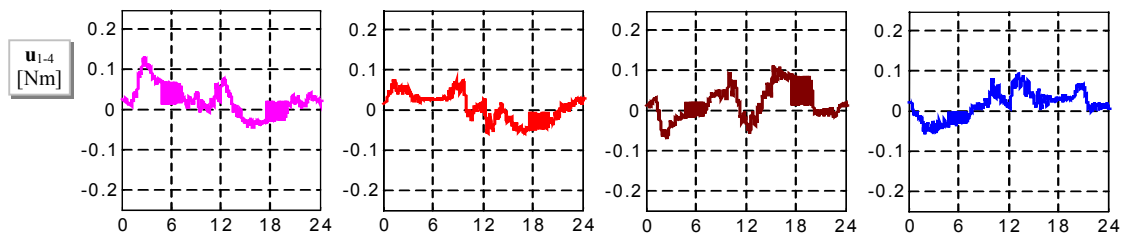


Fig. 5. Time history of real-time run of Predictive Control. (horizon  $N = 10$ , penalization  $\lambda = 10^{-10}$ , Sampling  $T_s = 0.01$  s; slow trajectory)

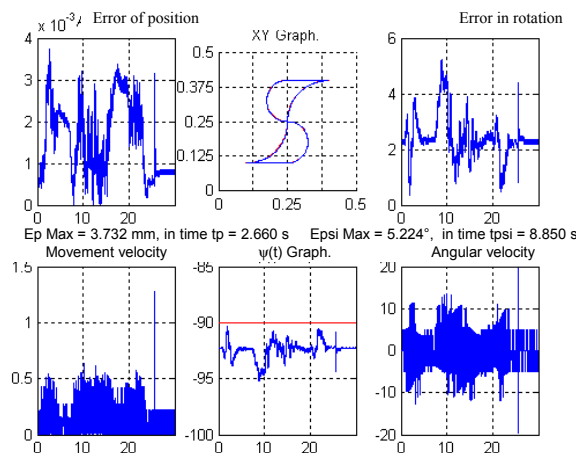


Fig. 6. Qualitative characterizations of real-time run of Predictive Control.

The last figure (Fig. 6) shows trends of errors. The errors are higher than in simulative case, because the real robot system includes drives and gears, which are not directly considered in mathematical model (2).

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