STATE-SPACE GENERALIZED PREDICITVE CONTROL FOR REDUNDANT PARALLEL ROBOTS

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Abstract:

The paper deals with the design and properties of Generalized Predictive Control for path control of the redundant parallel robots. It summarizes classical and root minimization of the quadratic criterion and direct and two-step design of actuators respectively. As an example, the planar redundant parallel robot is used. Moreover, the paper presents several possibilities to use Predictive control for compliance of some additional requirements as smooth trends of actuators or fulfillment antibacklash condition.

1. Introduction

Topically, the next development in industrial area is constrained by deficit of powerful machines with proportional dynamics and stiffness. At the same time, the new control techniques, which would be able to achieve higher accuracy with keeping of dexterity of the robot constructions, are missing or, on the other hand, there is no interest for their real application during research and development of new machines.

One of the promising ways of solving mentioned problems is utilization of new robot type based on parallel construction [1]. However, this new concept of the constructions brings new questions, especially in control area, thus the parallel structures give the possibilities to significantly improve mechanical parameters of new machines (dexterity, dynamics, stiffness, kinematics' accuracy etc.).

The aim of this paper is investigation of one potential control approach - Generalized Predictive Control (GPC) [2, 3] as an example of high-level model-based control. This approach firstly offers to achieve higher accuracy (better compliance with technological requirements; i.e. for robots: better compliance of planned trajectory) and at the same time effective cooperation of all actuators – drives. Secondly it offers several possibilities to realize some additional requirements [4], e.g., requirement on smooth trends of actuators – drives or fulfillment antibacklash condition can be mentioned.

The paper initially focuses on model description of the parallel structure, then continues with introduction of predictive control technique and finally shows simulation examples and briefly discuses real - time application.

2. Description of the robot model

The robot (manipulator) is a multibody system, which can be described by Lagrange's equations, in redundant case, of mixed type. These equations lead to the DAE system (the Differential - Algebraic Equations) in the following form:

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{s}} - \boldsymbol{\Phi}_s^T \boldsymbol{\lambda} &= \mathbf{g} + \mathbf{T}\mathbf{u} \\ \mathbf{f}(\mathbf{s}(t)) &= \mathbf{0} \end{aligned} \tag{1}$$

where $\mathbf{M}$ is a mass matrix, $\mathbf{s}$ is a vector of physical coordinates (their number is higher than number of degrees of freedom /DOF/), $\boldsymbol{\Phi}_s$ is an overall Jacobian of the system, $\boldsymbol{\lambda}$ are Lagrange's multipliers, $\mathbf{g}$ is a vector of right sides, matrix $\mathbf{T}$ transforms the inputs $\mathbf{u}$ (n torques) into n drives and $\mathbf{f}(\mathbf{s}(t)) = \mathbf{0}$ represents geometrical constrains.

The physical coordinates $\mathbf{s}$ consist of the independent coordinates $\mathbf{x}$ (Cartesian coordinates of the fix point of the cutting tool or gripper), drives'- actuators' coordinates $\mathbf{q}_1$ and other auxiliary geometrical coordinates $\mathbf{q}_2$.

Let us consider the possibility to transform the model (1) into independent coordinates $\mathbf{x}$ [5]. As follows, the DAE robot model is transformed to the ordinary differential model (ODE). It means that the Lagrange's multipliers disappear and design of the robot control becomes considerably simpler. Then the final model of the robot system is the following:

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{x}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{x}} = \mathbf{R}^T\mathbf{g} + \mathbf{R}^T\mathbf{T}\mathbf{u} \tag{2}$$

It is very important to note, firstly, that the Jacobian matrix $\mathbf{R}$ is the basis of the null space of the overall Jacobian $\boldsymbol{\Phi}_s$ and thus it satisfies the expression

$$\boldsymbol{\Phi}_s\ \mathbf{R} = \mathbf{R}^T\boldsymbol{\Phi}_s^T = \mathbf{0} \ \text{ and } \ \dot{\mathbf{s}} = \mathbf{R}\dot{\mathbf{x}} \rightarrow \ddot{\mathbf{s}} = \mathbf{R}\ddot{\mathbf{x}} + \dot{\mathbf{R}}\dot{\mathbf{x}} \tag{3}$$

and, secondly, the Jacobian $\mathbf{R}$ can be decomposed into submatrixes $\mathbf{R}_{\mathbf{q}_1}$, $\mathbf{R}_{\mathbf{q}_2}$ and $\mathbf{R}_\mathbf{x} = \mathbf{I}_\mathbf{x}$. Submatrix $\mathbf{R}_{\mathbf{q}_1}$ ($= (\mathbf{R}^T\mathbf{T})^T$) defines important relation between $\dot{\mathbf{q}}_1$ and $\dot{\mathbf{x}}$ as

$$\dot{\mathbf{q}}_1 = \mathbf{R}_{\mathbf{q}_1}\dot{\mathbf{x}} \quad \left(\equiv \frac{d\mathbf{q}_1}{dt} = \mathbf{R}_1 \cdot \frac{d\mathbf{x}}{dt} \right) \tag{4}$$

which will be useful in section dealing with design of control law in root form. $\mathbf{R}_1$ can be also obtained from geometrical relation $\mathbf{q}_1(\mathbf{x})$:

$$\mathbf{R}_1 = \left[\frac{\partial \mathbf{q}_1(\mathbf{x})}{\partial x_1}, \cdots, \frac{\partial \mathbf{q}_1(\mathbf{x})}{\partial x_n} \right] \Bigg|_{\substack{n = \text{number of independent} \\ \text{coordinates } = \\ = \text{ degrees of freedom}}} \tag{5}$$

3. Classical design of control law

The principal task of control of the robots is accomplishment of their movement along a planned trajectory (technological requirements). In some cases, it is very sophisticated and difficult for general control approaches like classical PID structures. Therefore, the new control approaches, which are being developed, are directly adjusted for concrete system (machine, robot). High-level controls, which use knowledge of the mathematical model e.g. (1, 2), represent suitable approach, which takes into account dynamic trend of the controlled system. In this way, it can better comply with mentioned requirements from technology. On the basis of the dynamic model, equation (2), high level controls globally optimize whole control process and can predict future actions. One of them is Generalized Predictive Control (GPC).

The Predictive control [2, 4] is a multi-step control based on local optimization of the quadratic criterion, where the linearized equation or state formula is used (i.e. only the nearest future control signal is evaluated). This approach admits combination of feedback~feedforward parts.

For design of predictive control law, the nonlinear model (2) must be linearized [5] and converted from continuous to discrete time. This described model transformation enables us to consider the classical discrete state formula in the following form:

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k) \\ \mathbf{x}(k) &= \mathbf{C}\,\mathbf{X}(k) \end{aligned} \tag{6}$$

where $\mathbf{X}$ is composed as $\mathbf{X} = [\mathbf{x}, \dot{\mathbf{x}}]^T$ and $\mathbf{x}$ agrees with equation (2). Furthermore for law derivation, the expression of new unknown output values $\mathbf{x}$ from topical state $\mathbf{X}$ is needed. The following lines imply this expression.

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{C}\mathbf{X}(k) \\ \hat{\mathbf{X}}(k+1) &= \mathbf{A}\ \mathbf{X}(k) + \mathbf{B}\mathbf{u}(k) \\ \hat{\mathbf{x}}(k+1) &= \mathbf{C}\mathbf{A}\ \mathbf{X}(k) + \mathbf{C}\mathbf{B}\mathbf{u}(k) \\ &\vdots \\ \hat{\mathbf{X}}(k+N) &= \mathbf{A}^N\mathbf{X}(k) + \mathbf{A}^{N-1}\mathbf{B}\mathbf{u}(k) + \cdots + \mathbf{B}\mathbf{u}(k+N-1) \\ \hat{\mathbf{x}}(k+N) &= \mathbf{C}\mathbf{A}^N\mathbf{X}(k) + \mathbf{C}\mathbf{A}^{N-1}\mathbf{B}\mathbf{u}(k) + \cdots + \mathbf{C}\mathbf{B}\mathbf{u}(k+N-1) \end{aligned}$$

then the prediction of $\mathbf{x}$ is the following

$$\hat{\mathbf{x}} = \mathbf{f} + \mathbf{G}\,\mathbf{u} \tag{7}$$

$$\mathbf{f} = \begin{bmatrix} \mathbf{CA} \\ \vdots \\ \mathbf{CA}^N \end{bmatrix} \mathbf{X}(k) \quad \text{and} \quad \mathbf{G} = \begin{bmatrix} \mathbf{C} & \mathbf{B} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{CA}^{N-1}\mathbf{B} & & \cdots & \mathbf{CB} \end{bmatrix} \tag{8}$$

Now we can optimize the quadratic criterion. The criterion is optimized in instant k, with using predictions of $\mathbf{x}$ ($\hat{\mathbf{x}} = [\hat{\mathbf{x}}_{k+1} \cdots \hat{\mathbf{x}}_{k+N}]^T$)

$$\begin{aligned} J_k &= \mathcal{E}\left\{(\hat{\mathbf{x}} - \mathbf{w})^T (\hat{\mathbf{x}} - \mathbf{w}) + \mathbf{u}^T \boldsymbol{\lambda}\, \mathbf{u}\right\} = \\ &= \mathcal{E}\left\{(\mathbf{G}\mathbf{u} + \mathbf{f} - \mathbf{w})^T (\mathbf{G}\mathbf{u} + \mathbf{f} - \mathbf{w}) + \mathbf{u}^T \boldsymbol{\lambda}\, \mathbf{u}\right\} \end{aligned} \tag{9}$$

where $\mathcal{E}$ is an operator of mean value, N is a horizon of prediction, $\mathbf{x}$ is a vector of outputs, $\mathbf{w}$ are desired values, $\boldsymbol{\lambda}$ is a penalization of input and $\mathbf{u}$ is a vector of robot inputs. Considering the condition of optimization

$$J_k \overset{!}{=} \min \tag{10}$$

for criterion (9), the resultant control law is

$$\mathbf{u} = (\mathbf{G}^T\mathbf{G} + \boldsymbol{\lambda})^{-1}\mathbf{G}^T\ (\mathbf{w} - \mathbf{f}) \tag{11}$$

This control law (11) can be already used. It should be noted that only the first element $\mathbf{u}(k)$ from vector $\mathbf{u}$ is used. If penalization $\boldsymbol{\lambda}$ is greater than zero, then the matrix $\mathbf{G}^T\mathbf{G}$ is regular for all cases, adequately actuated even for redundant cases. Theoretical case of zero penalization $\boldsymbol{\lambda}$ with redundant robot can be solved by pseudoinversion [6].

4. Design of control law in root form

This chapter aims on derivation of control law for different configuration of elements in mathematical model (2), which needs matrices with smaller dimensions. Moreover, if the penalization is positive, the computation also holds the redundant properties (if exist). It can be also used for accomplishment of additional control requirements.

Furthermore, in this chapter, the advantages of the root optimization of quadratic criterion (9) are used, marked out by compact notation and good preparation for operations with huge matrices.

Let us proceed from nonlinear differential model (2) and from its simplified form:

$$\begin{aligned} \mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{y}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{y}} &= \mathbf{R}^T\mathbf{g} + \mathbf{R}^T\mathbf{T}\mathbf{u} \\ \mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{y}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{y}} &= \mathbf{R}^T\mathbf{g} + \mathbf{FM} \end{aligned} \tag{12}$$

where new vector **FM** represents new fictitious input to the system so called general forces.

In equation (12) we can apply the same procedures of linearization [5], discretization and use the same composition of prediction formula (chapter 3, $\hat{\mathbf{x}} = \mathbf{f} + \mathbf{G}\ \mathbf{u}$, (7)) for future output values.

The quadratic criterion (9) ($J_k = \mathcal{E}\left\{(\hat{\mathbf{x}} - \mathbf{w})^T (\hat{\mathbf{x}} - \mathbf{w}) + \mathbf{u}^T \boldsymbol{\lambda}^T \boldsymbol{\lambda}\, \mathbf{u}\right\}$) can be rewritten in the root form as a product of matrices

$$J_k = \left[\, [\hat{\mathbf{x}} - \mathbf{w}]^T,\ \mathbf{u}^T \right] \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\lambda}^T \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\lambda} \end{bmatrix} \begin{bmatrix} [\,\hat{\mathbf{x}} - \mathbf{w}] \\ \mathbf{u} \end{bmatrix} = \mathbf{J}^T \mathbf{J} \qquad (13)$$

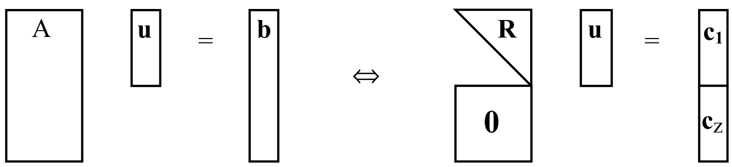
Now we can work only with root of the criterion

$$\mathbf{J} = \begin{bmatrix} \hat{\mathbf{x}} \\ \boldsymbol{\lambda}\,\mathbf{u} \end{bmatrix} - \begin{bmatrix} \mathbf{w} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{G}\mathbf{u} + \mathbf{f} \\ \boldsymbol{\lambda}\,\mathbf{u} \end{bmatrix} - \begin{bmatrix} \mathbf{w} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{G} \\ \boldsymbol{\lambda} \end{bmatrix} \mathbf{u} + \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{w} \\ \mathbf{0} \end{bmatrix} [= \mathbf{0}] \qquad (14)$$

$$\mathbf{A} \quad \mathbf{u} - \quad \mathbf{b} \quad = \mathbf{0} \qquad (15)$$

and consecutively we look for such action **u**, which minimizes root form (14, 15), i.e. we look for **u**, in order to minimize the norm |**J**|. If we annul the root of criterion (14), we will obtain system of equation (15) with more rows than columns (over-determined system).

For computation, the triangular-orthogonal decomposition [6] is used. It reduces excess rows of the matrix **A** $[(2{\cdot}N{\cdot}n)\times(N{\cdot}n)]$ and elements of vector **b** $[2{\cdot}N{\cdot}n]$ (n is a number of DOF) into upper triangle **R** and shorter vector $\mathbf{c}_1$ according to the following scheme:



(16)

Vector $\mathbf{c}_Z$ is a loss vector. Its Euclidean norm $|\mathbf{c}_Z|$ is equal to root of quadratic criterion; scalar $\sqrt{J}$ (i.e. $J = \mathbf{c}_Z^T \mathbf{c}_Z$).

For solution, we need only the upper part of the system of equations (16), which can be simply solved in view of the vector of actuators **u** by backward-run procedure.

Obtained actuators represent fictitious generalized force effects **u**, from which only the first subvector (for k instant) $\mathbf{u}(k) = \mathbf{FM}$ is used. It must be recomputed, according to substitution in equations (12), to really used actions (drives):

$$\mathbf{R}^T \mathbf{T}\, \mathbf{u}_{(\text{drives})} = \mathbf{FM} \qquad (17)$$

with the same meaning of matrices **R** and **T** as in the system of differential equations (12). System (17) generally expresses deficient rank equation system (lower number of rows than columns i.e. than unknown real inputs - actions). There is again possibility to use pseudo-inverse of the matrix $\mathbf{R}^T\mathbf{T}$ there.

5. Examples and Conclusions

This section shows different actuators' trends for different control requirements, applied on planar redundant parallel robot (*Figure 1.*), for one selected trajectory.

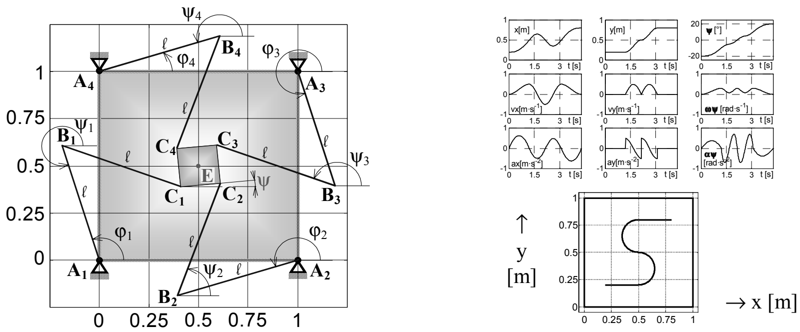
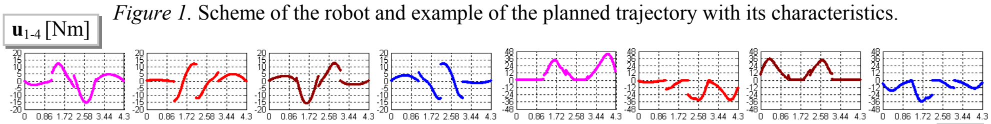


Figure 1. Scheme of the robot and example of the planned trajectory with its characteristics.



t [s]

Figure 2. Trends of four actuators unlimited and limited by antibacklash condition (sampling Ts = 0.01s; max. error 1 μm; penalization $\lambda = 10^{-12}$; horizon $N = 10$).

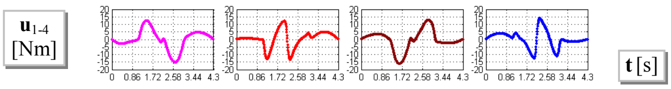


Figure 3. Smoothing of the actuators trends for trajectory from *Figure 1.* – variation of penalization (sampling Ts=0.01s; max. error 2.02 mm; penalization $\lambda = 5\cdot10^{-8}$; horizon $N = 10$).

The second, root approach is suitable for real application, because it represents less mathematical operations than classical approach. At present, it is tested on real laboratory model with the same structure as in *Figure 1*. As for result, both the approaches, classical and root control designs, are identical.

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6. References

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