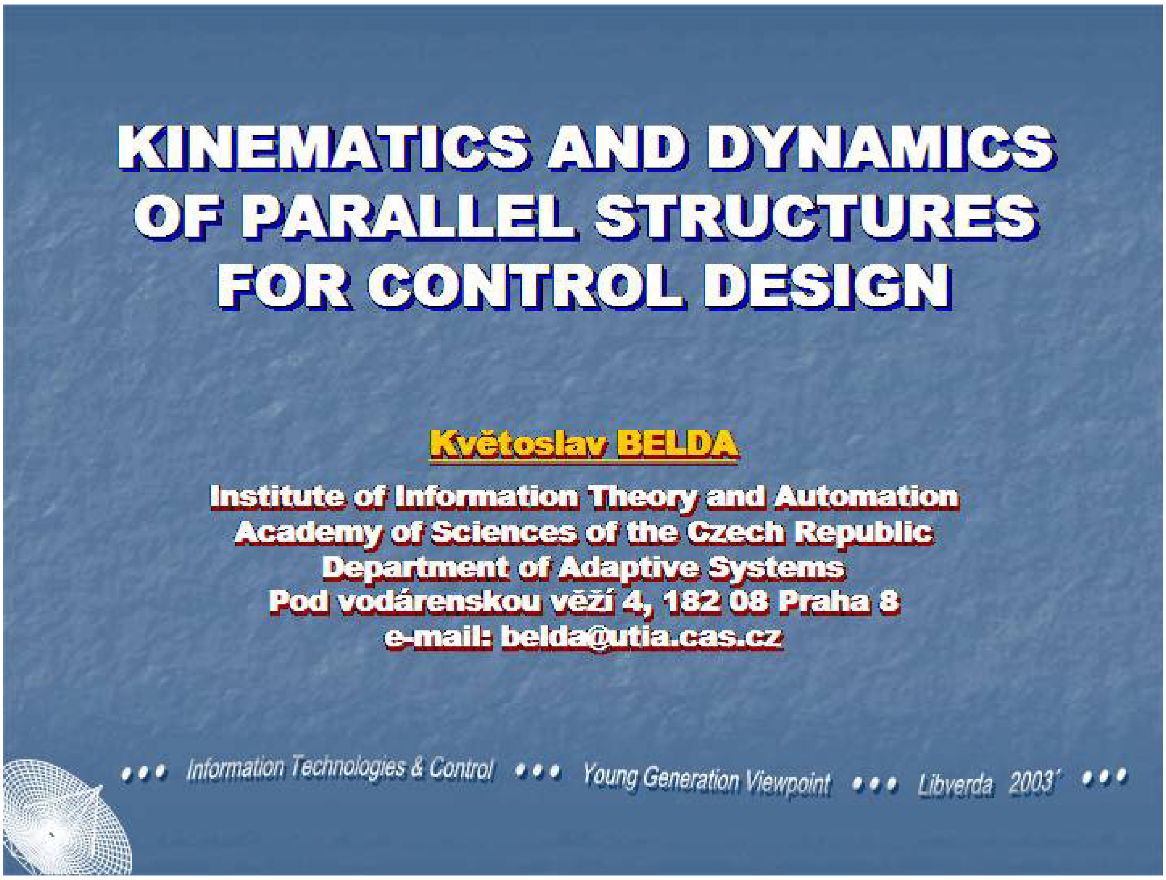
KINEMATICS AND DYNAMICS
OF PARALLEL STRUCTURES
FOR CONTROL DESIGN
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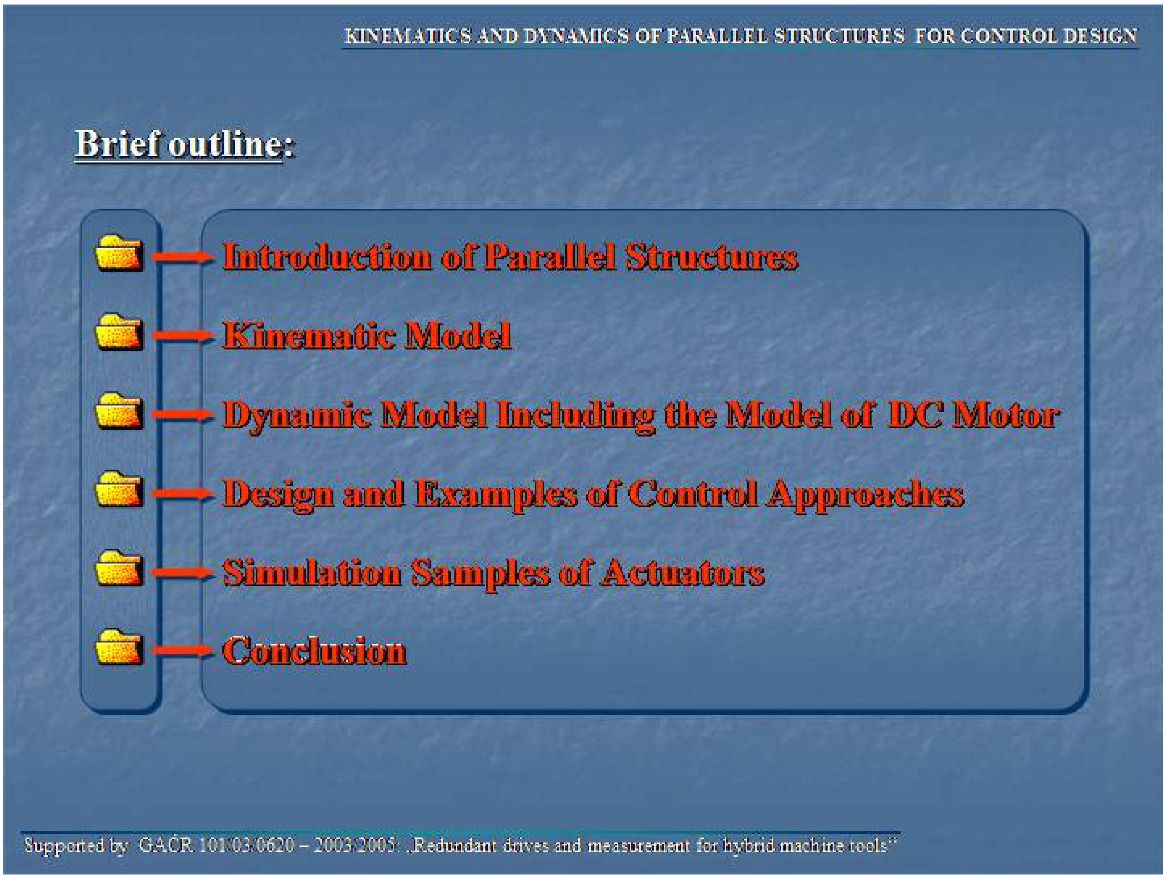
Information Technologies & Control
Young Generation Viewpoint
Libverda 2003'

1


KINEMATICS AND DYNAMICS OF PARALLEL STRUCTURES FOR CONTROL DESIGN
Brief outline:
Introduction of Parallel Structures
Kinematic Model
Dynamic Model Including the Model of DC Motor
Design and Examples of Control Approaches
Simulation Samples of Actuators
Conclusion
Supported by GAČR 101/03/0620 – 2003/2005: „Redundant drives and measurement for hybrid machine tools"

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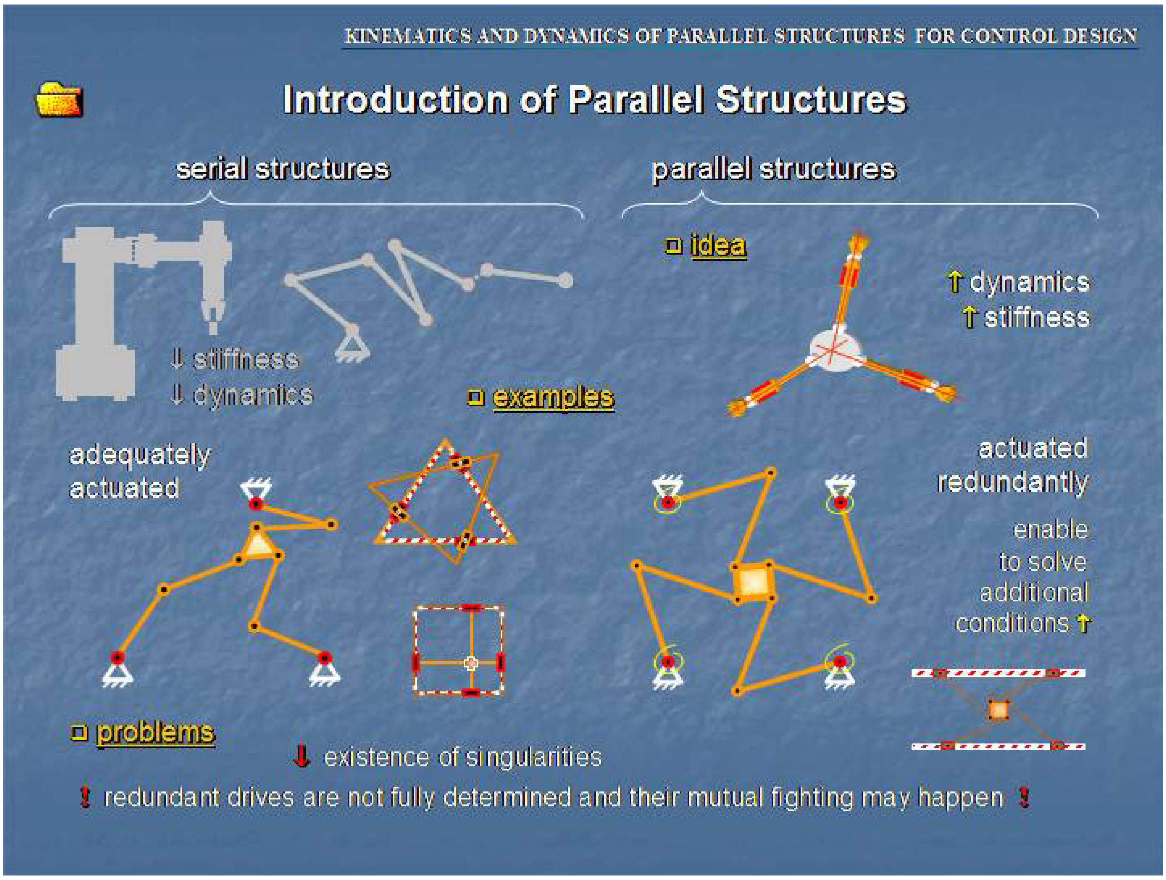
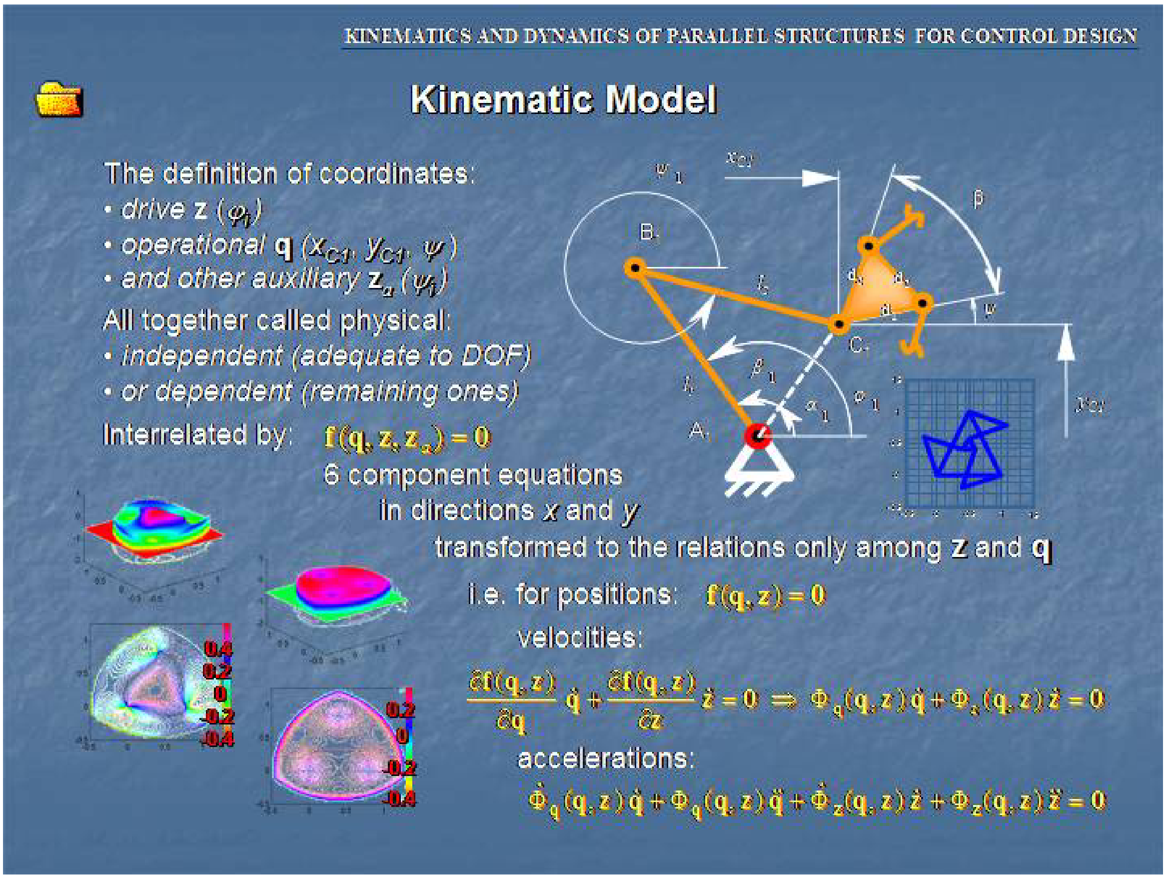
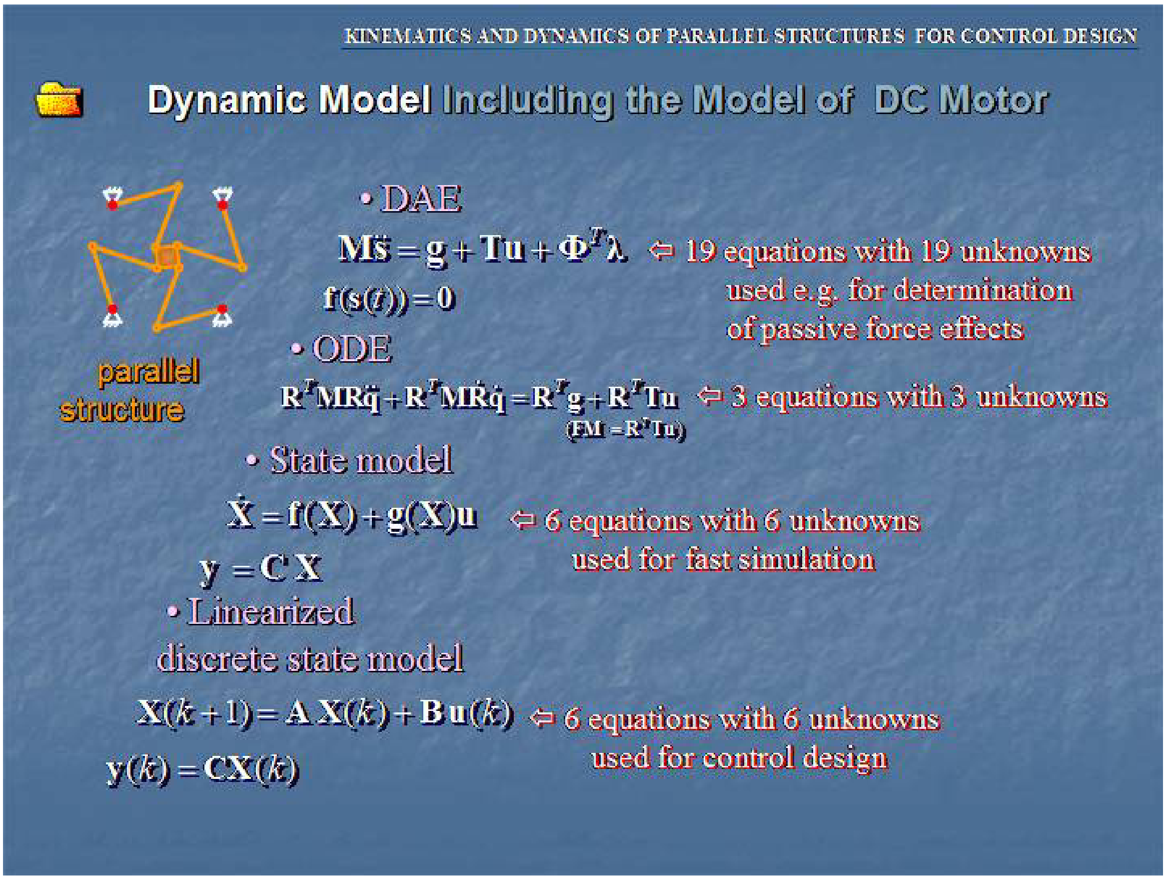

KINEMATICS AND DYNAMICS OF PARALLEL STRUCTURES FOR CONTROL DESIGN
Introduction of Parallel Structures
serial structures
parallel structures
idea
↑ dynamics
↑ stiffness
⇓ stiffness
⇓ dynamics
examples
adequately actuated
actuated redundantly
enable to solve additional conditions ↑
problems
existence of singularities
redundant drives are not fully determined and their mutual fighting may happen

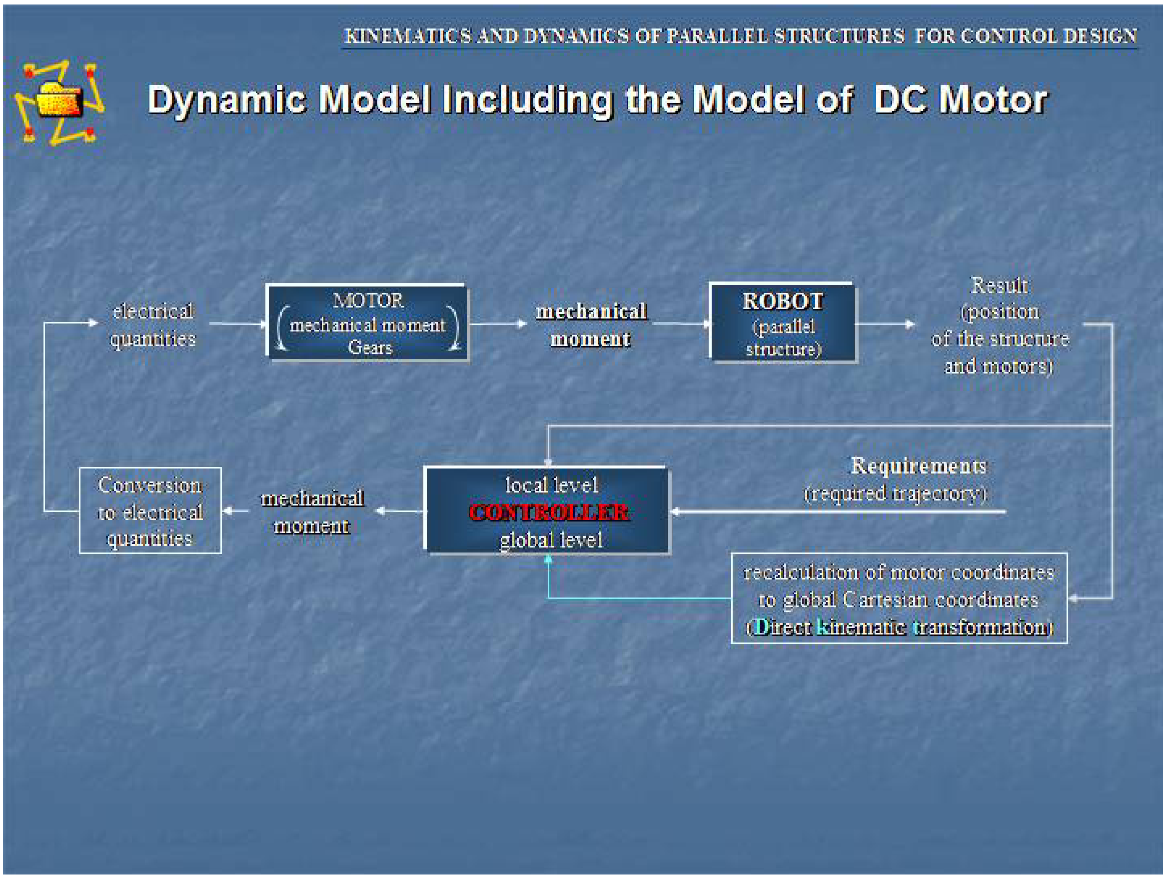
3


KINEMATICS AND DYNAMICS OF PARALLEL STRUCTURES FOR CONTROL DESIGN
Kinematic Model
The definition of coordinates:
• drive z (φ_i)
• operational q (x_{C1}, y_{C1}, ψ)
• and other auxiliary z_a (ψ_i)
All together called physical:
• independent (adequate to DOF)
• or dependent (remaining ones)
Interrelated by: $f(q, z, z_a) = 0$
6 component equations
in directions x and y
transformed to the relations only among z and q
i.e. for positions: $f(q, z) = 0$
velocities:
$\frac{\partial f(q,z)}{\partial q}\dot{q} + \frac{\partial f(q,z)}{\partial z}\dot{z} = 0 \Rightarrow \Phi_q(q,z)\dot{q} + \Phi_z(q,z)\dot{z} = 0$
accelerations:
$\dot{\Phi}_q(q,z)\dot{q} + \Phi_q(q,z)\ddot{q} + \dot{\Phi}_z(q,z)\dot{z} + \Phi_z(q,z)\ddot{z} = 0$
β
ψ
B_i
C_i
A_i
x_{C1}
y_{C1}
0.4
0.2
0
-0.2
-0.4

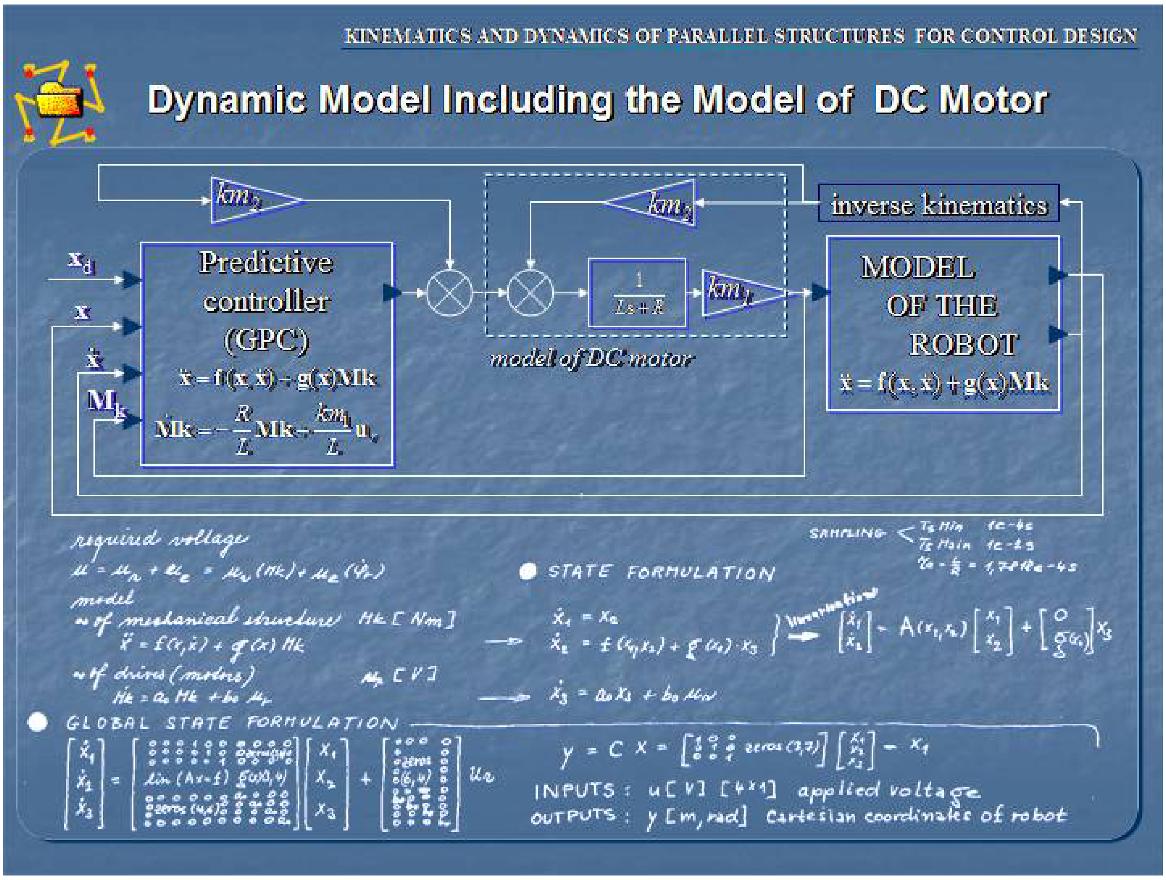
4


KINEMATICS AND DYNAMICS OF PARALLEL STRUCTURES FOR CONTROL DESIGN
Dynamic Model Including the Model of DC Motor
parallel structure
• DAE
$\mathbf{M}\ddot{\mathbf{s}} = \mathbf{g} + \mathbf{T}\mathbf{u} + \mathbf{\Phi}^T\lambda$
$\mathbf{f}(\mathbf{s}(t)) = 0$
⇦ 19 equations with 19 unknowns used e.g. for determination of passive force effects
• ODE
$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{q}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{q}} = \mathbf{R}^T\mathbf{g} + \mathbf{R}^T\mathbf{T}\mathbf{u}$
$(\mathbf{FM} = \mathbf{R}^T\mathbf{T}\mathbf{u})$
⇦ 3 equations with 3 unknowns
• State model
$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}) + \mathbf{g}(\mathbf{X})\mathbf{u}$
$\mathbf{y} = \mathbf{C}\,\mathbf{X}$
⇦ 6 equations with 6 unknowns used for fast simulation
• Linearized discrete state model
$\mathbf{X}(k+1) = \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k)$
$\mathbf{y}(k) = \mathbf{C}\mathbf{X}(k)$
⇦ 6 equations with 6 unknowns used for control design

5


KINEMATICS AND DYNAMICS OF PARALLEL STRUCTURES FOR CONTROL DESIGN
Dynamic Model Including the Model of DC Motor
electrical quantities
MOTOR (mechanical moment Gears)
mechanical moment
ROBOT (parallel structure)
Result (position of the structure and motors)
Conversion to electrical quantities
mechanical moment
local level CONTROLLER global level
Requirements (required trajectory)
recalculation of motor coordinates to global Cartesian coordinates (Direct kinematic transformation)

6


KINEMATICS AND DYNAMICS OF PARALLEL STRUCTURES FOR CONTROL DESIGN
Dynamic Model Including the Model of DC Motor
km2
km2
inverse kinematics
xd
x
ẋ
Mk
Predictive controller (GPC)
ẍ = f(x,ẋ) + g(x)Mk
Ṁk = −R/L Mk + km1/L u
1/(Ls+R)
km1
model of DC motor
MODEL OF THE ROBOT
ẍ = f(x,ẋ) + g(x)Mk
required voltage
model
STATE FORMULATION
GLOBAL STATE FORMULATION
INPUTS : u [V] [4x1] applied voltage
OUTPUTS : y [m, rad] Cartesian coordinates of robot

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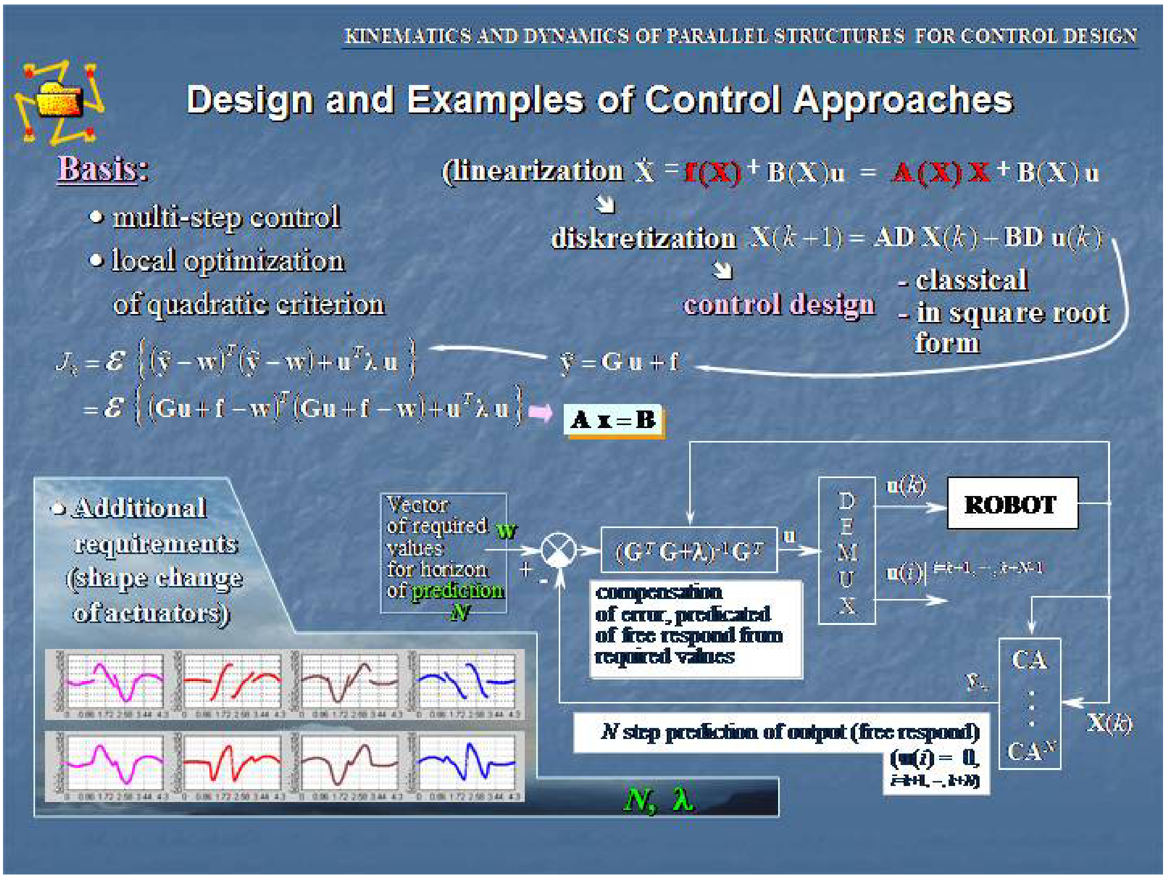
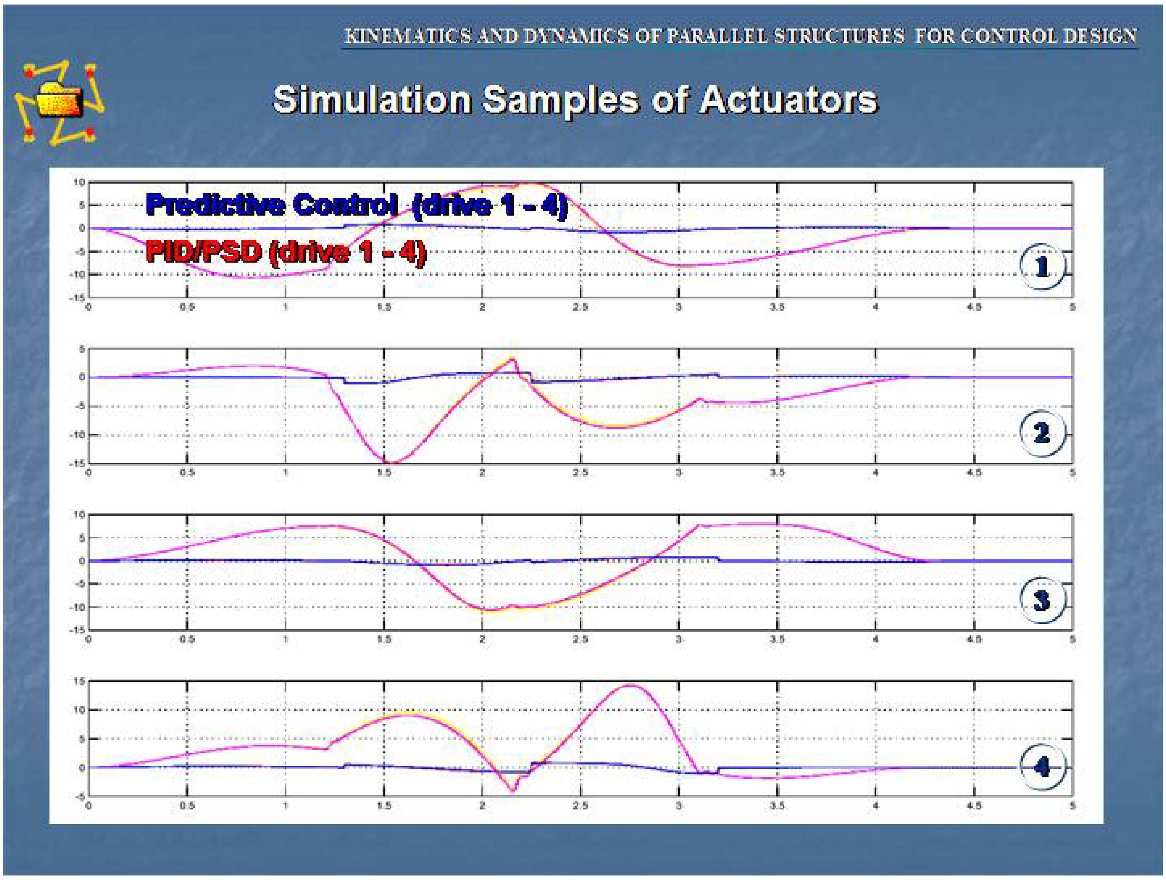
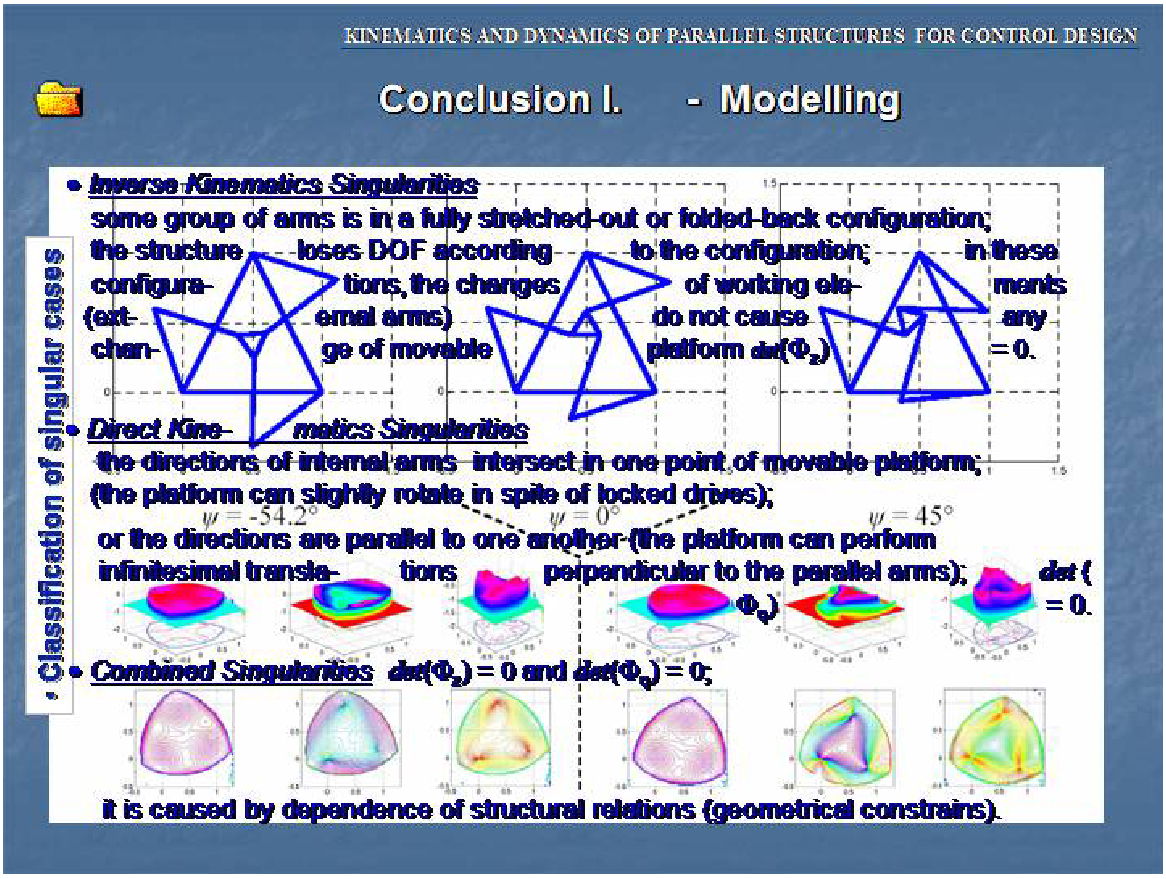

KINEMATICS AND DYNAMICS OF PARALLEL STRUCTURES FOR CONTROL DESIGN
Design and Examples of Control Approaches
Basis:
• multi-step control
• local optimization of quadratic criterion
(linearization Ẋ = f(X) + B(X)u = A(X)X + B(X)u
diskretization X(k+1) = AD X(k) + BD u(k)
control design
- classical
- in square root form
ŷ = G u + f
A x = B
• Additional requirements (shape change of actuators)
Vector of required values for horizon of prediction N
w
(GᵀG+λ)⁻¹Gᵀ
u
DEMUX
u(k)
ROBOT
compensation of error, predicated of free respond from required values
CA
CA^N
X(k)
N step prediction of output (free respond)
N, λ

8


KINEMATICS AND DYNAMICS OF PARALLEL STRUCTURES FOR CONTROL DESIGN
Simulation Samples of Actuators
Predictive Control (drive 1 - 4)
PID/PSD (drive 1 - 4)
1
2
3
4

9


KINEMATICS AND DYNAMICS OF PARALLEL STRUCTURES FOR CONTROL DESIGN
Conclusion I. - Modelling
Classification of singular cases
Inverse Kinematics Singularities
some group of arms is in a fully stretched-out or folded-back configuration;
the structure loses DOF according to the configuration; in these configurations, the changes of working elements (external arms) do not cause any change of movable platform det(Φz) = 0.
Direct Kinematics Singularities
the directions of internal arms intersect in one point of movable platform;
(the platform can slightly rotate in spite of locked drives);
or the directions are parallel to one another (the platform can perform infinitesimal translations perpendicular to the parallel arms); det(Φq) = 0.
ψ = -54.2°
ψ = 0°
ψ = 45°
Combined Singularities det(Φz) = 0 and det(Φq) = 0;
it is caused by dependence of structural relations (geometrical constrains).

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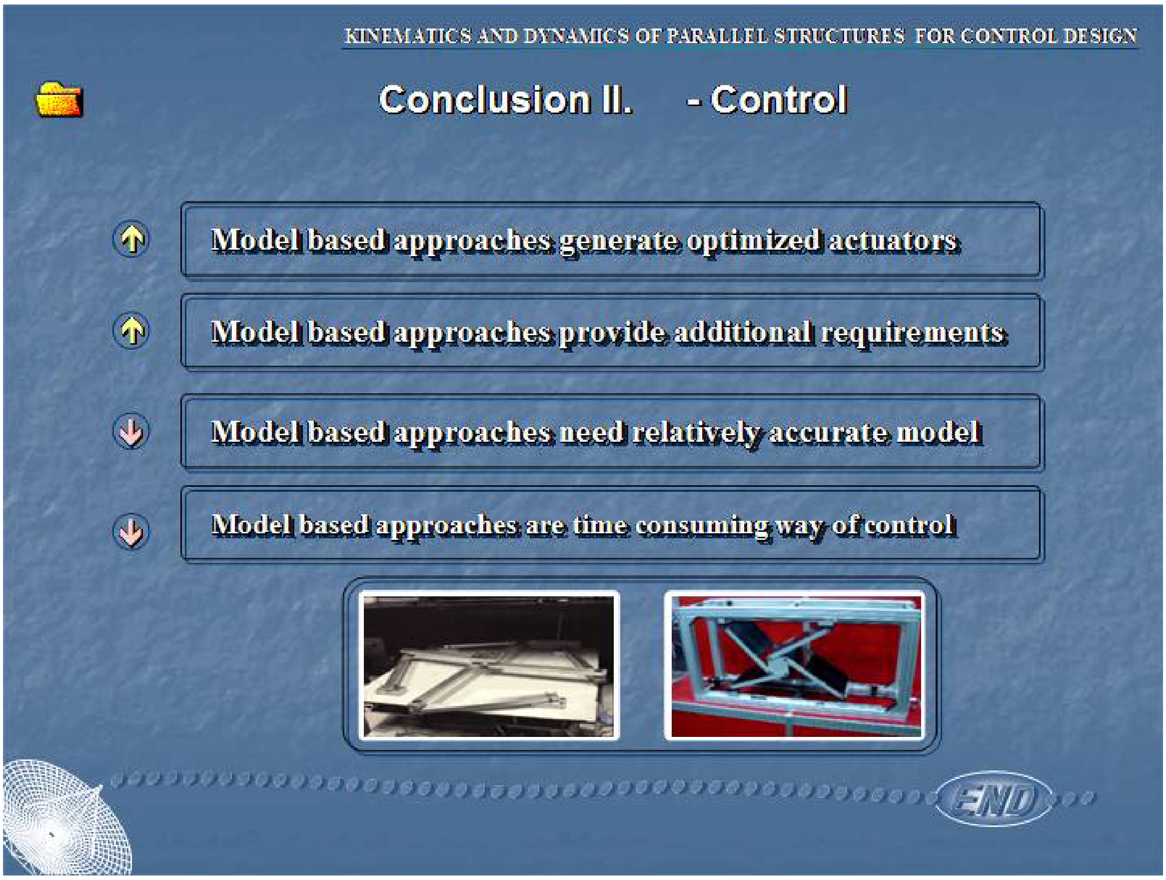
KINEMATICS AND DYNAMICS OF PARALLEL STRUCTURES FOR CONTROL DESIGN
Conclusion II. - Control
Model based approaches generate optimized actuators
Model based approaches provide additional requirements
Model based approaches need relatively accurate model
Model based approaches are time consuming way of control
END