STUDY OF CONTROL OF PLANAR REDUNDANT PARALLEL ROBOT

JOSEF BÖHM, KVĚTOSLAV BELDA, MICHAEL VALÁŠEK
Institute of Information Theory and Automation,
Academy of Sciences of the Czech Republic,
Pod vodárenskou věží 4, 182 08 Praha 8, Czech Republic.
Emails: bohm@utia.cas.cz, belda@utia.cas.cz

Abstract: Industrial robots and manipulators are being constantly developed in order to improve their accuracy and speed. Parallel robots seem to be the promising way of the solution to this problem. This paper deals with the design and simulation of control of one such robot. Generalized Predictive Control (GPC), Inverse Dynamics Control (IDC) and Sliding Mode Control (SMC) are discussed here. The main reasons for their choice were that these control approaches are suitable for adaptation on redundant case and they can be successfully implemented for nonlinear systems and used for control in real time.

Keywords: Planar redundant parallel robot, Drives, Control (GPC, IDC, SMC), Nonlinear system.

I. INTRODUCTION

The most topical industrial robots and manipulators do not cope with increasing requirements on speed and accuracy. This is caused mainly by the limitation of the acceleration given by their construction. Therefore, new approaches of their construction are being found, in order to allow performance of these requirements. Parallel robots seem to be one of the promising ways, how to solve this problem [2]. And, moreover, they have several advantages over traditional serial robots:

- All or almost all drives are located on the basic frame i.e. drives do not move with the robot and they do not have any hold on the moving mass and stiffness.
- Truss construction of the robots leads to higher stiffness than in serial types. It is advantageous for accurate machining and positioning.

On the other hand, the parallel robots have also several disadvantages:

- Their workspace is mostly smaller than for serial ones and includes the singular positions where the robot loses controllability. They can be removed by the redundant actuation as in our case.
- Collisions of arms with the platform are more probable than in the case of actual robots. This must be taken into account at the planning of desired trajectory.

As an example, let us consider one such redundantly actuated planar parallel robot (Fig.1), which will be presented here.

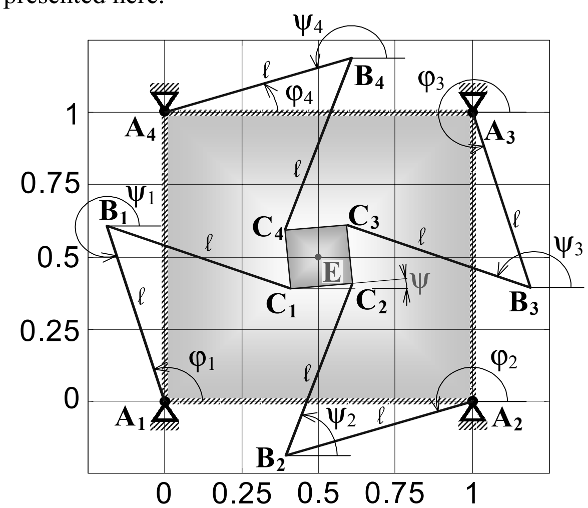

Fig. 1. Scheme of planar parallel robot with the most important geometrical description (The coordinates of center E of movable platform and its angle of winding ψ).

It consists of the basic frame, which at the same time encloses workspace of the robot, four independent drives, movable platform and eight arms, which connect the movable platform with the basic frame. The arms are parallelly situated.

From the mechanical point of view, the robot has one drive and one pair of arms redundant, because generally the number of degrees of freedom of body in a plane is only three (movement in the direction of axis x, movement in the direction of axis y and angle of winding). Accordingly, for controlling the robot and for its mechanical determination, only three pairs of appropriate arms are necessary. But in this case, the singular position in workspace will appear. Therefore a redundant drive is used in order to overcome the problem. And, moreover, it improves stiffness and rotation speed of movable platform and gives the possibility to comply with the other additional control requirements.

The aim of this paper is to investigate the way how to provide the control of overactuated parallel robot and to try to satisfy the specific performance requirements like anti-backlash control within possible control approaches.

II. MODEL OF THE ROBOT

From the control point of view, the principal task of a robot is its movement along a planned trajectory. For this type of the robot, it is usually given in Cartesian coordinates, which are very efficacious for users. In these coordinates, the robot motion can be described by the following nonlinear differential equation [2]:

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{y}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{y}} - \mathbf{R}^T\mathbf{g} = \mathbf{R}^T\mathbf{T}\mathbf{u} \tag{1}$$

it can be rewritten in the state formula in this form:

$$\begin{aligned}\dot{\mathbf{X}}(t) &= \mathbf{f}(\mathbf{X}) + \mathbf{g}(\mathbf{X})\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{h}\,\mathbf{X}(t)\end{aligned} \tag{2}$$

The input variables are torques of all drives. The state variables are coordinates of center **E** (x_E , y_E) of movable platform, its angle of winding (ψ) and their derivative. The output variables are only a few selected from state variables, i.e. coordinates of the center (x_E , y_E , ψ).

The functions **f**(**X**), **g**(**X**) are highly nonlinear reflecting the kinematics structure of the parallel robot. The non-linearity stems from nonlinear dependence of the center coordinates on the coordinates of drives, where torques are applied.

The other possibility is the description in joints coordinates [3], which is applied in classical constructions, but for our case it would be more difficult.

III. APPROACHES TO CONTROL

The control should ensure the best possible compliance of the trajectory and at the same time effective cooperation between the necessary drives and one redundant drive. As a solution several approaches have been chosen, namely, discrete Generalized Predictive Control (GPC) [1], continuous Inverse Dynamics Control (IDC) [3] and discrete Sliding Mode Control (SMC) [4]. The main reasons for their choice are the following:

- The model of robot is well known.
- All named controls can be modified to reflect various control requirements (minimum energy, torques of one sign - anti-backlash etc.).
- Their algorithmization is relatively simple and it can be easily implemented in computer.

Now they can be separately introduced.

- PREDICTIVE CONTROL (GPC)

The Predictive control [1] is a multi-step control based on local optimization of the quadratic criterion, where the linarized equation or state formula can be used (i.e. only the nearest future control signal is used). The approach admits combination of feedback~feedforward parts.

As mentioned above, for the quadratic criterion the nonlinear model (2) must be linearized [2] and converted from continuous to discrete time. After that we can consider discrete state formula in this form:

$$\begin{aligned}\mathbf{X}(k+1) &= \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k)\\ \mathbf{y}(k) &= \mathbf{C}\,\mathbf{X}(k)\end{aligned} \tag{3}$$

where sense of **X** and **y** are the same as in (2). The base of predictive control is the expressing of new unknown output values **y** from actual topical state **X**. The following rows imply it.

$$\begin{aligned}
\mathbf{y}(k) &= \mathbf{C}\,\mathbf{X}(k)\\
\mathbf{X}(k+1) &= \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k)\\
\hat{\mathbf{y}}(k+1) &= \mathbf{CA}\,\mathbf{X}(k) + \mathbf{C}\,\mathbf{B}\,\mathbf{u}(k)\\
&\vdots\\
\hat{\mathbf{X}}(k+N) &= \mathbf{A}^N\mathbf{X}(k) + \mathbf{A}^{N-1}\mathbf{B}\,\mathbf{u}(k) + \cdots + \mathbf{B}\,\mathbf{u}(k+N-1)\\
\hat{\mathbf{y}}(k+N) &= \mathbf{CA}^N\mathbf{X}(k) + \mathbf{CA}^{N-1}\mathbf{B}\,\mathbf{u}(k) + \cdots + \mathbf{CB}\,\mathbf{u}(k+N-1)
\end{aligned} \tag{4}$$

prediction of **y** is then following $\Rightarrow\ \hat{\mathbf{y}} = \mathbf{G}\,\mathbf{u} + \mathbf{f}$ (5a)

$$\text{where}\ \mathbf{G} = \begin{bmatrix}\mathbf{C} & \mathbf{B}\cdots & 0\\ \vdots & \ddots & \vdots\\ \mathbf{CA}^{N-1}\mathbf{B} & \cdots & \mathbf{CB}\end{bmatrix}\ \text{and}\ \mathbf{f} = \begin{bmatrix}\mathbf{CA}\\ \vdots\\ \mathbf{CA}^N\end{bmatrix}\mathbf{X}(k) \tag{5b}$$

Now we can optimize quadratic criterion at certain instant k using predictions of $\mathbf{y}$ ($\hat{\mathbf{y}} = [\hat{\mathbf{y}}_{k+1}\cdots\hat{\mathbf{y}}_{k+N}]^T$)

$$\begin{aligned}J_k &= \mathcal{E}\left\{(\hat{\mathbf{y}} - \mathbf{w})^T(\hat{\mathbf{y}} - \mathbf{w}) + \mathbf{u}^T\lambda\,\mathbf{u}\right\} =\\ &= \mathcal{E}\left\{(\mathbf{G}\,\mathbf{u} + \mathbf{f} - \mathbf{w})^T(\mathbf{G}\,\mathbf{u} + \mathbf{f} - \mathbf{w}) + \mathbf{u}^T\lambda\,\mathbf{u}\right\}\end{aligned} \tag{6}$$

where $\mathcal{E}$ is operator of mean value, N is horizon of prediction, **y** is vector of outputs, **w** are desired values, λ is penalization of input and **u** is vector of robot inputs.

Condition is $J_k \overset{!}{=} \min \Rightarrow \mathbf{u} = \left(\mathbf{G}^T\mathbf{G} + \lambda\right)^{-1}\mathbf{G}^T(\mathbf{w} - \mathbf{f})$ (7)

This control law can be already used. It must be noted that only the first element $\mathbf{u}_k$ from vector **u** is used. If penalization λ is greater than zero, the matrix $\mathbf{G}^T\cdot\mathbf{G}$ is regular and the problem with redundant action disappears. For constraint of actuators the quadratic programming is used.

Graphical representation of system with Predictive control is in Fig.2.

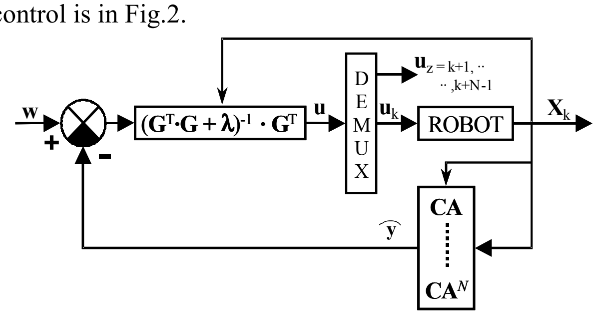

Fig. 2. Control circuit with GPC and parallel robot.

- INVERSE DYNAMICS CONTROL (IDC)

Consider the robot described by nonlinear differential equation (1) $\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{y}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{y}} - \mathbf{R}^T\mathbf{g} = \mathbf{R}^T\mathbf{T}\mathbf{u}$, which can be rewritten (for simplification) as follows

$$\begin{aligned}\ddot{\mathbf{y}} - \left(\mathbf{R}^T\mathbf{M}\mathbf{R}\right)^{-1}\left(\mathbf{R}^T\mathbf{g} - \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{y}}\right) &= \left(\mathbf{R}^T\mathbf{M}\mathbf{R}\right)^{-1}\mathbf{R}^T\mathbf{T}\mathbf{u} \\ \ddot{\mathbf{y}} - \qquad \mathbf{f}(\mathbf{y},\dot{\mathbf{y}}) \qquad &= \qquad \mathbf{B}(\mathbf{y}) \qquad \mathbf{u}\end{aligned} \tag{8}$$

Note the matrix $\mathbf{R^TMR}$ is a square regular matrix. It ensues from mechanical interpretation.

The approach (IDC) is based on the idea to find a control vector **u** as a function of system state. The classical approach [3] assumes that matrix **B(y)** is a full rank matrix which can be inverted. After we can obtain control law as a function of robot state in the form:

$$\mathbf{u} = \left(\mathbf{B}(\mathbf{y})\right)^{-1}\ddot{\mathbf{y}} + \left(\mathbf{B}(\mathbf{y})\right)^{-1}\mathbf{f}(\mathbf{y},\dot{\mathbf{y}}) \tag{9}$$

Such control leads to finding stabilizing control law for system described by $\mathbf{q} = \ddot{\mathbf{y}}$

$$\mathbf{u} = \widetilde{\mathbf{B}}(\mathbf{y})\mathbf{q} + \mathbf{n}(\mathbf{y},\dot{\mathbf{y}}) \tag{10}$$

where **q** represents a new input vector to all robot control circuit. The nonlinear control law eq. (10) is termed as inverse dynamics control since it includes computation of robot inverse dynamics itself. The system with this control is linear with respect to the new input q.

In view of eq. (10) the choice:

$$\mathbf{q} = -\mathbf{K}_P\mathbf{y} - \mathbf{K}_D\dot{\mathbf{y}} + \mathbf{r} \tag{11}$$

leads to the linear system of second-order equations:

$$\ddot{\mathbf{y}} + \mathbf{K}_D\dot{\mathbf{y}} + \mathbf{K}_P\mathbf{y} = \mathbf{r} \tag{12}$$

which, on the assumption that positive definite diagonal matrices $\mathbf{K_P}$ = diag.$\{\mathbf{\Omega^2}\}$ and $\mathbf{K_D}$ = diag.$\{\mathbf{2\zeta\Omega}\}$, is stable. **(** $\ddot{y} + 2\zeta\,\dot{y} + \Omega^2 y = 0$ the analogy with math model of free damping of mechanical systems, where Ω represents natural frequency and ζ is damping ratio**)**.

Eq. (10) considering eq. (11) represents global linearization of system dynamics. The eq. (12) may be also written for desired values $\mathbf{y_d}$(t):

$$\ddot{\mathbf{y}}_d + \mathbf{K}_D\dot{\mathbf{y}}_d + \mathbf{K}_P\mathbf{y}_d = \mathbf{r} \tag{13}$$

Subtracting eq. (12) from eq. (13) gives the homogeneous second-order differential equation:

$$\ddot{\widetilde{\mathbf{y}}} + \mathbf{K}_D\dot{\widetilde{\mathbf{y}}} + \mathbf{K}_P\widetilde{\mathbf{y}} = \mathbf{0}, \quad \widetilde{\mathbf{y}} = \mathbf{y}_d - \mathbf{y} \tag{14}$$

expressing the dynamics of position aberration (error) during tracking the desired trajectory (desired values).

Block diagram of a system with described Inverse dynamics control is in Fig.3.

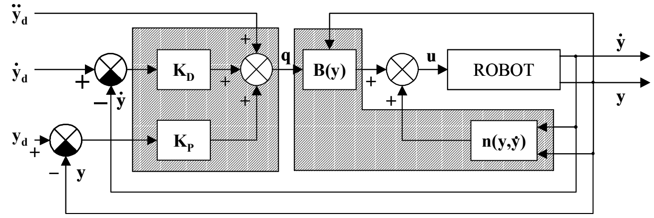

Fig. 3. Control circuit with IDC and robot.

Now we must come back to the case when the matrix is singular as in our case. It is caused by redundant action and the size of matrix **B** is 3x4 because output vector **y** has three elements $[x,y,\psi]^T$ and input vector **u** consists of four actuators. This problem is solved by orthogonal-triangular decomposition (The Least square methods for deficient rank problem [5]). This result gives possibility to compute a control law, let us say actuators, with minimum energy or using free redundant element to perform requirement on anti-backlash control.

- SLIDING MODE CONTROL (SMC)

Nowadays a lot of controls are implemented by digital computers, but the most systems have continuous character described by differential equations. That is why this discordance must be solved. The systems must be transformed into discrete form, used for the design of controller.

Here presented discrete type of Sliding mode control [4] is derived analogically to the theory of stability in a continuous domain. Generally it is based on the 'switching' control action and the performance of Lyapunov theorem of stability conditions. The state is driven towards a desired switching (sliding) hyperplane under Lyapunov control. The 'switching' maintains the state on this hyperplane once it is reached, in spite of perturbations. This method offers an advantage of accuracy at the cost of control dithering, which ensues from the 'switching' part.

Let us consider the nonlinear state formula (2) without output equation $\quad \dot{\mathbf{X}}(t) = \mathbf{f}(\mathbf{X}) + \mathbf{g}(\mathbf{X})\mathbf{u}(t) \qquad (15)$

It can be simply discretized by Taylor series with sampling period δ in the following form:

$$\mathbf{X}(k+1) = \mathbf{A}(\mathbf{X}(k)) + \mathbf{B}(\mathbf{X}(k))\mathbf{u}(k) \tag{16}$$

where

$$\begin{aligned}\mathbf{A}(\mathbf{X}(k))_{\left(\substack{i=1:2,\\ j=1:3}\right)} &= \begin{bmatrix} x(k)_{\substack{i=1,\\ j=1:3}} + \delta\; x(k)_{\substack{i=2,\\ j=1:3}} + \dfrac{\delta^2}{2} f_{j=1:3}(\mathbf{X}(k)) \\ x(k)_{\substack{i=2,\\ j=1:3}} + \delta\; f_{j=1:3}(\mathbf{X}(k)) \end{bmatrix} \\ \mathbf{B}(\mathbf{X}(k))_{\left(\substack{i=1:2,\\ j=1:3}\right)} &= \begin{bmatrix} \dfrac{\delta^2}{2} g_{j=1:3}(\mathbf{X}(k)) \\ \delta\; g_{j=1:3}(\mathbf{X}(k)) \end{bmatrix}\end{aligned} \tag{17}$$

and $\mathbf{X}(k) = [x(k)_{1\text{-}6}]^T = [x_E(k)\; y_E(k)\; \psi(k)\; \dot{x}_E(k)\; \dot{y}_E(k)\; \dot{\psi}(k)]^T$.

Now we can start with derivation of control law. Firstly let us define a discrete Lyapunov function:

$$V(\mathbf{X}(k)) = \mathbf{s}^2(\mathbf{X}(k)) > 0 \tag{18}$$

then $$\Delta\mathbf{V}(\mathbf{X}(k)) = \mathbf{s}^2(\mathbf{X}(k+1)) - \mathbf{s}^2(\mathbf{X}(k)) \le 0 \tag{19}$$

represents the discrete attractivity condition. It can be rewritten as follows into two inequalities (20) (21)

$$\mathbf{s}^2(\mathbf{x}(k+1)) \le \mathbf{s}^2(\mathbf{x}(k)) \Rightarrow |\mathbf{s}(k+1)| \le |\mathbf{s}(k)| \quad / \cdot sign\,(\mathbf{s}(k))$$

$$|\mathbf{s}(k+1) \cdot sign\,(\mathbf{s}(k))| \le |\mathbf{s}(k) \cdot sign\,(\mathbf{s}(k))| = \mathbf{s}(k) \cdot sign\,(\mathbf{s}(k))$$

we solve two cases :

1. $\mathbf{s}(k+1) \cdot sign\,(\mathbf{s}(k)) > 0 \Rightarrow sign\,(\mathbf{s}(k+1)) = sign\,(\mathbf{s}(k))$

   $\mathbf{s}(k+1) \cdot sign\,(\mathbf{s}(k)) \le \mathbf{s}(k) \cdot sign\,(\mathbf{s}(k))$

$$\Rightarrow (\mathbf{s}(k+1) - \mathbf{s}(k)) \cdot sign(\mathbf{s}(k)) \le 0 \tag{20}$$

2. $\mathbf{s}(k+1) \cdot sign\,(\mathbf{s}(k)) < 0 \Rightarrow sign\,(\mathbf{s}(k+1)) \ne sign\,(\mathbf{s}(k))$

   $-\mathbf{s}(k+1) \cdot sign\,(\mathbf{s}(k)) \le \mathbf{s}(k) \cdot sign\,(\mathbf{s}(k))$

$$\Rightarrow (\mathbf{s}(k+1) + \mathbf{s}(k)) \cdot sign(\mathbf{s}(k)) \ge 0 \tag{21}$$

Eq. (21) and (22) are an attractivity condition and a convergence condition, respectively. According to them the **s** dynamics may be chosen as

$$\mathbf{s}(k+1) = e^{-P\delta}\mathbf{s}(k) - \mathbf{K}sign(\mathbf{s}(k)) \tag{22}$$

where P is positive scalar, δ is sampling and **K** is positive diagonal matrix. Then eq. (20) and (21) are satisfied. (It is also possible to choose **s** dynamics with opposite signs.) Now, we define the sliding hypersurfaces as:

$$\mathbf{s}(k) = \mathbf{C}\mathbf{e}(k) \tag{23}$$

where $\mathbf{s}(k) = [s_1(k)\ s_2(k)\ \cdots\ s_m(k)]^T$, $\mathbf{e}(k) = \mathrm{X}(k) - \mathrm{X}_d(k)$. $\mathbf{X}_d(k)$ is a vector of desired values with the same size as a vector $\mathrm{X}(k)$, $\mathbf{C} = diag(\mathbf{C}^i)$, $\mathbf{C}^i = [c_1^i\ c_2^i\ \cdots\ c_n^i]$. $\mathbf{C}^i$ is chosen in order to satisfy Jury's stability condition of discrete systems, i is state variable index and n is order of system. At this moment we can write **s** at time t=(k+1) δ:

$$\mathbf{s}(\mathrm{k}+1) = \\ = \mathbf{C}\mathbf{e}(\mathrm{k}+1) = \mathbf{C}[\mathbf{A}(\mathrm{k}) + \mathbf{B}(\mathrm{k})\mathbf{u}(\mathrm{k}) + \Psi(\mathrm{k}) - \mathbf{X}_d(\mathrm{k}+1)] \tag{24}$$

Using (22), the discrete control law is obtained like this:

$$\mathbf{u}(k) = -(\mathbf{CB}(k))^{-1}\{\mathbf{C}[\mathbf{A}(k) + \Psi(k) - \mathbf{X}_d(k+1)] \\ - e^{-P\delta}\mathbf{s}(k) + \mathbf{K}sign(\mathbf{s}(k))\} \tag{25}$$

$\boldsymbol{\psi}(k)$ represents unknown perturbation in time k δ, which can be estimated by $\Psi(k-1)$

$$\Psi(k-1) = \mathbf{x}_{\substack{actual\\topical}}(k) - \mathbf{A}(k-1) - \mathbf{B}(k-1)\mathbf{u}(k-1) \tag{26}$$

This estimation process is valid provided the dynamics of perturbation are considerably slower than discretization frequency and moreover an order of perturbation magnitude is much smaller.

If (25) substitutes into (24), we obtain:

$$\mathbf{s}(k+1) = e^{-P\delta}\mathbf{s}(k) + \mathbf{C}[\Psi(k) - \Psi(k-1)] - \mathbf{K}sign(\mathbf{s}(k)) \tag{27}$$

And on this basis considering conditions (20), (21) and the fact that variations in perturbation are slow against sampling frequency, the diagonal elements of matrix **K** can be selected as:

$$[k_1\ k_2\ \cdots\ k_m]^T = \eta\ \mathbf{C}|\Psi(k-1)| > \mathbf{C}|\Psi(k) - \Psi(k-1)|, \\ \text{for} \quad \eta > 0 \tag{28}$$

Finally we must afresh answer to question of redundancy. In the text above we consider that the product of matrices $\mathbf{CB}(k)$ is regular and may be inverted, but the product in our case has again deficient rank. We can use the same solution as previous part – orthogonal-triangular decomposition – Least square method for deficient rank problem [5]. This result gives anew possibility to compute control law, let us say actuators, with minimum energy and using free redundant element to perform requirement on anti-backlash control.

IV. ILLUSTRATIVE EXAMPLES

For the simulation of the robot a plan of trajectory must be prepared and must be realizable for the robot. For example in our simulation a trajectory composed of bisector segments and arc segments was chosen. The trajectory was time-parameterized with constant period. That is the matter of choice. When planning trajectory we have considered kinematics laws as a relationship between acceleration, velocity and position, and, moreover, constrains of torques etc. This desired trajectory and its kinematic characterizations are shown in Fig.4 and Fig.5.

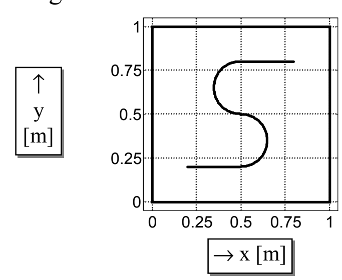

Fig.4. Desired trajectory for planar redundant robot.

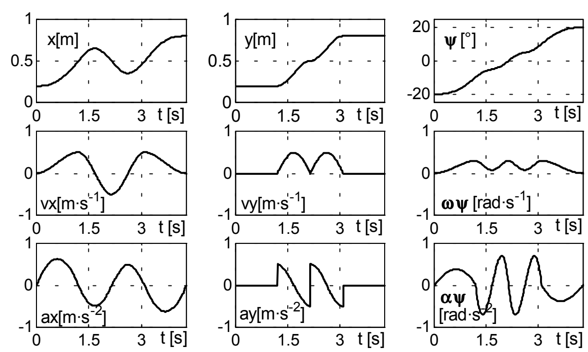

Fig.5. Kinematic characterizations.
(positions x, y, ψ, velocities and accelerations)

For the described trajectory above, the time history of four torques is shown, firstly for the control with minimum energy, secondly one figure deals with their comparison and finally, an example of anti-backlash will be introduced.

the penalization of actuator in quadratic criterion on higher value. The multi-step character of GPC provides smooth torques without alternating. This property is more needful for manipulators than for robots because at manipulator it depends only on final position.

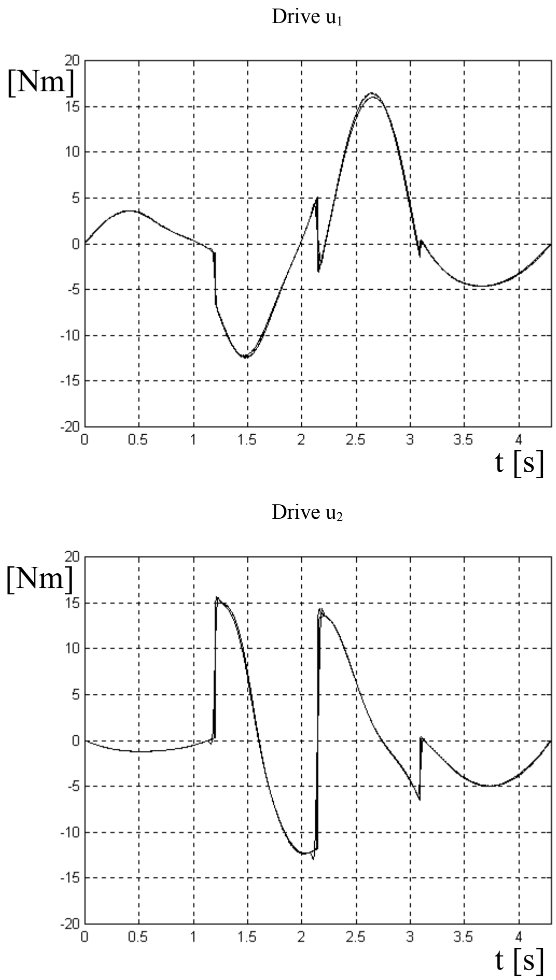

Fig. 6. The first and second torque-actuator.

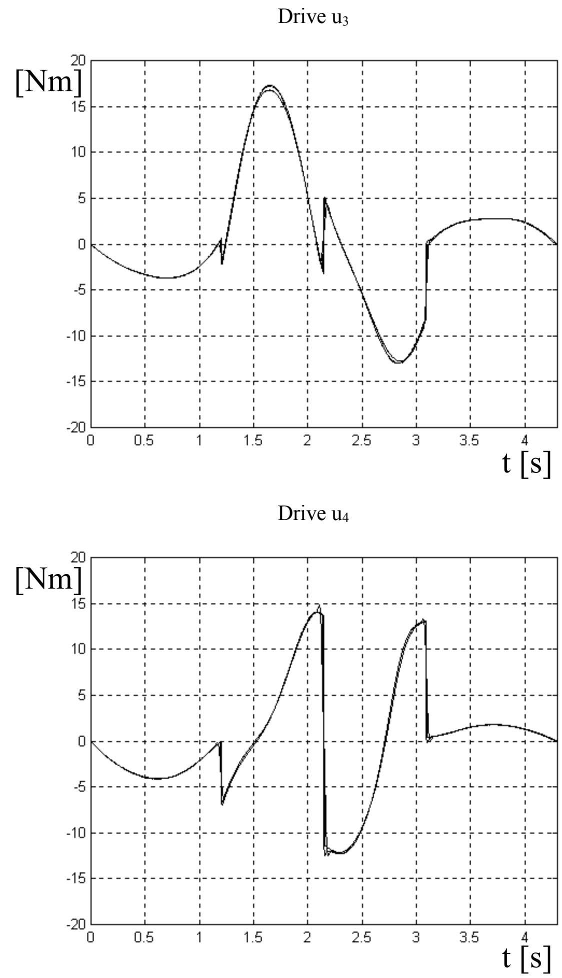

Fig. 7. The third and fourth torque-actuator.

The sharp jumps are caused by the change of kinematic characterizations of the trajectory. To be exact, there, where the bisector segment of the trajectory turns into arc segment or one arc segment changes into another arc segment, the change of kinematic properties happens.

From kinematics point of view the components of velocities and accelerations change, but against these, their common resulting velocities have constant magnitude only with modified direction.

All presented control approaches have smooth trend (except for points with change of segment character). It is good for drives.

At predictive control approach there is a possibility to influence smoothness of torques-actuators. If we wish to have a smooth torques, and it does not depend on accurate compliance of the planed trajectory, we can reset

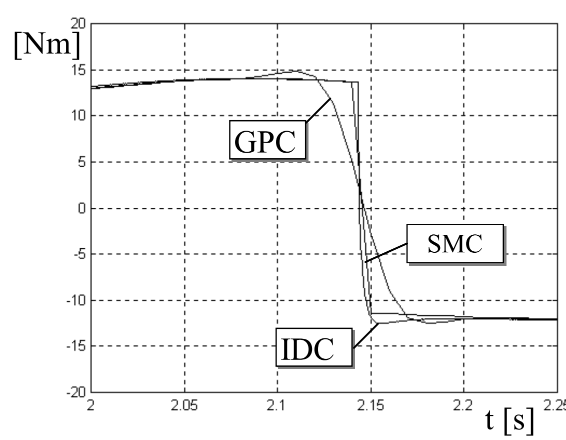

Fig. 8. Zoom of fourth drive u_4 at time 2 – 2.25s
for all presented control approaches.

In Fig.8. the multi-step character of GPC is perceptible well (Note: Setting of horizon of GPC is N = 10 steps).

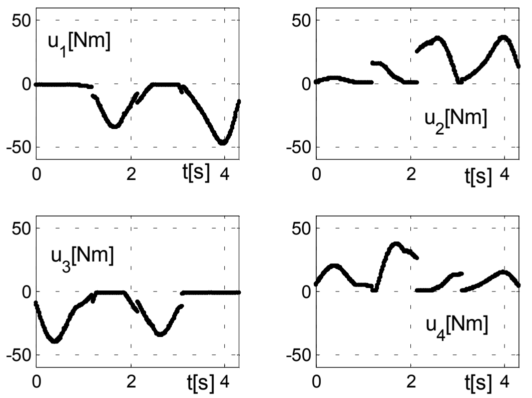

Fig. 9. An example of torques of anti-backlash control.

And finally, Fig.10.-12. show some quality results of presented controls.

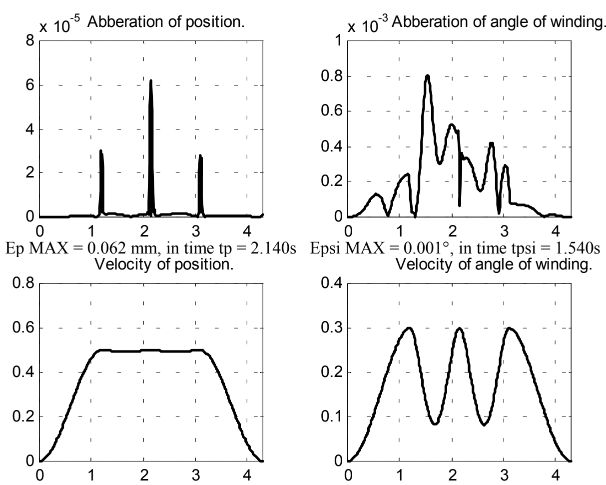

Fig. 10. Aberrations of trajectory GPC.

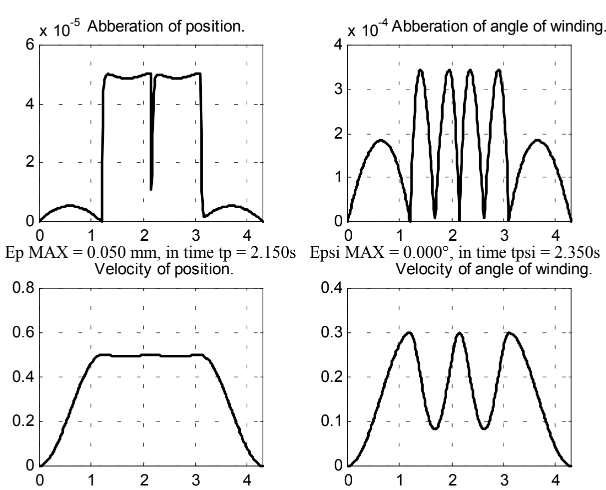

Fig. 11. Aberrations of trajectory IDC.

In Fig. (10), (11) and (12), the actual process of velocity of position and velocity of winding are shown. It corresponds with desired ones.

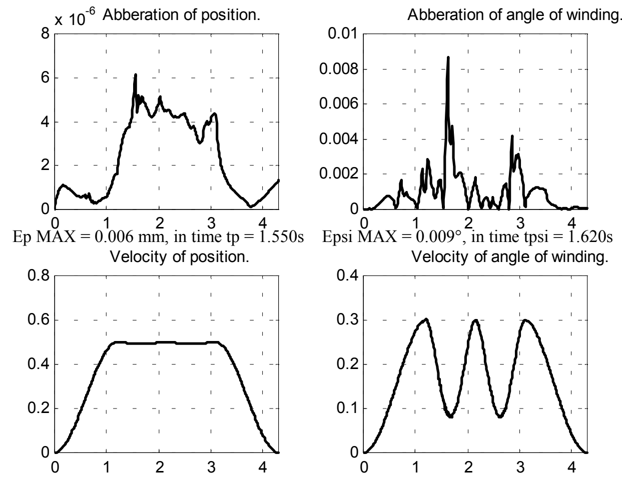

Fig. 12. Aberrations of trajectory with SMC.

These results have been obtained at setting of parameters of controls, described in Tab. 1.

Table of one setting of parameters of presented approaches
Generalized predictive control (GPC)
Penalization $\lambda = 10^{-10}$, Horizon N = 10.
Inverse dynamics control (IDC)
Diag. element of matrix $\mathbf{K_P}$ = 18000, diag. element of matrix $\mathbf{K_D}$ = 400
Sliding mode control (SMC)
Damping P=8, gain coefficient η=1.25, Jury's polynom $\mathbf{C}^i$ [110 10]

Tab. 1. Setting of parameters.

Controls of the robot were also tested with certain additional noise in range 10^{-4} mm (10% of average aberration). The effect did not markedly appear in trends of torques.

V. CONCLUSION

All approaches indicated here (GPC, IDC, SMC) are certainly a way to control given redundant parallel robot. Designed controls are suitable even for classical robots and their design arises from them. The tuning of parameters is not critical, but all controls pose claim on accuracy of the model of the robot.

REFERENCES

[1] A. Ordys and D. Clarke, A state - space description for GPC controllers. *INT. J. Systems SCI.*, vol.24, no.9, 1993, 1727 – 1744.

[2] M. Valášek and P. Steinbauer, Nonlinear control of multibody systems. *Euromech 404*, 1999, 437 – 444.

[3] L. Sciaviccio and B. Siciliano, *Modeling and control of robot manipulators* (New York: McGraw-Hill Co., 1996).

[4] H. Elmali and N. Olgac, Sliding mode control with perturbation estimation (SMCPE) : a new approach, *INT. J. CONTROL*, vol.56, no.4, 1992, 923 – 941.

[5] Ch. Lawson and R. J. Hanson, *Solving least squares problems* (Prentice-Hall, Inc., 1974).

Acknowledgement

This research is supported by GA ČR 101/99/0729, 1999-2001: "Redundant parallel robot and its control".